Salmon's Bayesian Bridge to Kuhn

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1 Introduction

Bayesianism is a school of thought that employs probability theory, especially modified forms of Bayes' Theorem, as a model for rational human thought. Wesley Salmon, a well-known philosopher in support of Bayesianism, realized an opportunity to apply such concepts in the picture of science painted by Kuhn. The goal of this paper is, after a brief introduction of Wesley Salmon, to give a brief but adequate overview of Bayesianism, including a working example of Bayesian Inference, and then to give a synopsis of why and where Salmon thinks Bayesianism is applicable in Kuhn's picture of science.

2 An Overview of Salmon

Wesley Salmon was a true heavyweight in philosophy, writing numerous papers and books on subjects including logic, the nature of space and time, rationality, and of course, probability and scientific realism. In his academic career, Salmon initially wanted to become a minister, but quickly became unenthralled and converted to philosophy. He received his masters in philosophy in 1947 from University of Chicago. He then went on to gain his PhD in philosophy in 1950 from UCLA. As a student of philosophy, he worked with the likes of Rudolf Carnap and Hans Reichenbach, who was his PhD adviser. He also served as faculty at Brown University, Indiana University Bloomington, University of Arizona, and University of Pittsburgh, where he succeeded Carl Hempel as the University Professor in the Philosophy Department.

Salmon was a strong supporter of scientific induction and Bayesianism. He believed induction was a core ingredient of science, invoking the use of probability in the justification of inductive logic. He called the strict, deductive, models of the hypothetico-deductive method effectively oversimplifications of the scientific method. Salmon claimed considerations on the plausibility of scientific theories were an integral part of science and that these considerations often impacted the choice of theories in competitive times. These plausibility considerations, it seems, have a nice interpretation in Bayesian views of scientific justification. These interpretations will be further explained in section 5, where they will be seen to quite cleanly overlap with the picture painted by Thomas Kuhn's *Structure of Scientific Revolutions*.

3 Bayes' Theorem & Bayesian Inference

Thomas Bayes was an English philosopher, mathematician, and minister that lived in the early 18th century. His work, *An Essay towards solving a Problem in the Doctrine of Chances*, in which he laid out basic conditional probability and what is now known commonly as Bayes' Theorem, was actually published two years after his death in 1763 with the help of his friend, Richard Price. Bayes' Theorem is an indisputable formula of conditional probability as it stands below.

$$P(A|B) = \frac{P(B|A) * P(A))}{P(B)}$$

Stating that the probability of something, here A, given something else, B, is exactly the probability of B given A times the probability of A, all divided by the probability of B. So, effectively, an event or existence is more likely in a certain context if that context is more likely when that event or existence is true. More eloquently put, ”the Theorem's central insight [is] that a hypothesis is confirmed by any body of data that its truth renders probable” [3]. As stated above, this is a purely mathematical statement of probabilities that is indisputably true, but some take it as a tool in areas where truth becomes more heavily guarded. We have particular interest in this more disputable region of application for the purposes of this paper. Specifically, *Bayesian Inference*, an epistemological approach that invokes a modified form of Bayes' Theorem in the determination of probable theories of explanation, especially in science.

Bayesian Inference uses modified forms of Bayes' Theorem in the updating of the probability of some theory being true. We use the specific modified form shown below.

$$P(T_1|E.B) = \frac{P(T_1|B)P(E|T_1)}{\sum_i P(T_i|B)P(E|T_i)}$$

Stating that the probability of some theory, here T_1, being true given some evidence, E, and the background information, B, is equal to the probability of some theory being true given the background times the probability of the new evidence occurring given the theory being true, all divided by the sum of the same expression for all the possible theories in total. The theories $T_1, ..., T_i$ are required to be exhaustive and mutually exclusive, as to guarantee one and only one is ultimately true.

Bayesianism, for our purposes, is the use of this Bayesian Inference in the updating of beliefs, as well as a general coherence of beliefs with the tenants of probability theory in total. These tenants require two other things. One, that the total sum of one's belief values for the explanation of some specific case must be one. And two, that the belief value of any individual hypothesis be between zero and one.

Bayesian probability interpretations thus seem to give probability values themselves a distinct role as the belief value of an ideal perfect agent, or the betting behavior of some unbiased and exquisitely rational party. This interpretation gives probability new power and terrain, allowing for the application of the tools of probability theory, such as Bayes' Theorem, to questions of rational belief and other non-empirical propositions. This use of probability theory as a system for rational belief is often defended by means of the Dutch Book argument, which elucidates that if the basic rules of probability are violated by one's beliefs, they will always lose in the net total if they bet true to all of those beliefs simultaneously.

Opponents to Bayesianism claim such Dutch Book arguments show only that probability theory's basic tenants are necessary for rational thought, not that Bayesian Inference is the uniquely 'right' way to update one's beliefs. Thus, there is room in the realm of philosophical standpoints to subscribe to the second component of Bayesianism, as we've defined it, but reject the notion of the first. One may agree that a rational belief system must fit the tenants of probability theory, but reject the requirement that one's beliefs must be updated via the modified form of Bayes' Theorem.

Below is given an example of Bayesian Inference, as we've defined it. It is written in a programming language called R, for reproducibility.

4 An Example of Bayesian Inference

Imagine a Universe is created by God. It is centered around one urn containing an infinite number of red and blue balls. A people live in this universe and have access to this urn. Religious texts exist, stating this urn either contains 50 percent blue and 50 percent red balls, or 65 percent blue and 35 percent red balls.

```
##hypothesis are for percentage of blue balls
hypothesis1<-h1<-.5
hypothesis2<-h2<-.65

## 1 stands for blue and 0 for red
ontology1<-c(1,1,1,1,1,0,0,0,0,0)
ontology2<-c(1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)
```

Now, suppose one local tribe's religious sect takes the probability of the urn containing 65 percent blue balls to be high and a corresponding low probability of it containing 50 percent red balls. Every night, the tribe draws thirteen balls from the urn and records their colors.

```
PT1B<-initialPh1<-.25
PT2B<-initialPh2<-.75

X<-13
```

Now the tribe in question has recently suffered severe droughts. In fact, the tribe has called their religion into question and have put their top philosophers on the case of determining whether their religion is correct about the balls in the urn. After some deliberation, the philosopher's elected return to recommend a Bayesian approach to the determination of the correct ratio of colors. They claim they will only need a calculator capable of doing basic arithmetic and performing calculations of the binomial theorem. With this calculator, they will then use a modified form of Baye's theorem, as shown below, to calculate the probability of the other hypothesis in question being true.

$$P(T_1|E.B) = \frac{P(T_1|B)P(E|T_1)}{P(T_1|B)P(E|T_1) + P(T_2|B)P(E|T_2)}$$

The binomial theorem will yield the probability of the evidence occurring given one of the theories, $P(E|T_{1,2})$, or specifically, the probability of drawing a certain selection of colors assuming some ontology of the urn. The initial probability of either hypothesis, $P(T_{1,2}|B)$, will be based on the religious beliefs of the tribe, here as defined as 0.25 for the belief that the urn is 50% blue balls, and as 0.75 for the belief that the urn is 65% blue balls.

```
# The chance of this day's drawing occurring given either hypothesis is computed by
## the function defined below.
PET<-as_mapper(~dbinom(x=..1, size=X, prob=..2))
```

```
## The beliefs of the tribe are updated by the function below.
PTE<-as_mapper(~..1*..2/(..1*..2+..3*..4)); PTE2<-as_mapper(~..3*..4/(..1*..2+..3*..4))
```

The tribe, destitute and looking for hope, follow the philosopher's advice, recording their results of drawings for one week and updating their beliefs according to the recommended Bayesian approach. Below is their reported drawings of blue balls from the urn, of the thirteen drawn daily.

```
MondayDrawing<-MD<-sum(sample(ontology1, replace=TRUE, size=X))
TuesdayDrawing<-TuD<-sum(sample(ontology1, replace=TRUE, size=X))
WednesdayDrawing<-WD<-sum(sample(ontology1, replace=TRUE, size=X))
ThursdayDrawing<-ThD<-sum(sample(ontology1, replace=TRUE, size=X))
FridayDrawing<-FD<-sum(sample(ontology1, replace=TRUE, size=X))
SaturdayDrawing<-SaD<-sum(sample(ontology1, replace=TRUE, size=X))
SundayDrawing<-SuD<-sum(sample(ontology1, replace=TRUE, size=X))

WeeksDrawing<-WeekD<-c(MD,TuD,WD,ThD,FD,SaD,SuD)
WeekD

## [1] 5 4 3 5 6 5 7
```

Below is a table and graph showing their belief in the hypothesis that the urn contains 50% blue balls over the week.

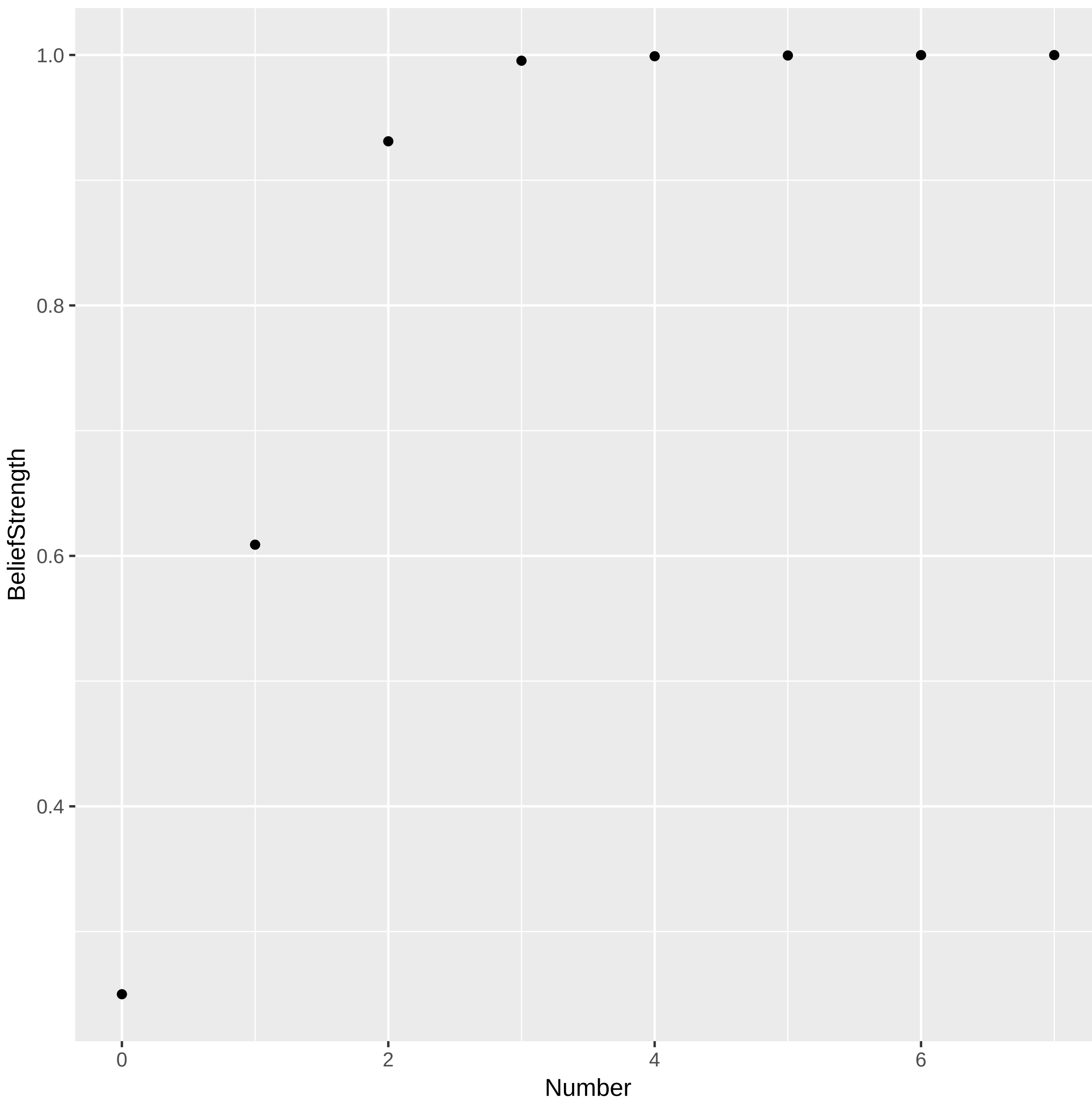
```
MondaysReportofLikely<-MPT1B<-PTE(PT1B,PET(MD,h1),PT2B,PET(MD,h2))
        MondaysReportofH2<-MPT2B<-PTE2(PT1B,PET(MD,h1),PT2B,PET(MD,h2))
TuesdaysReportofLikely<-TuPT1B<-PTE(MPT1B,PET(TuD,h1),MPT2B,PET(TuD,h2))
        TuesdaysReportofH2<-TuPT2B<-PTE2(MPT1B,PET(TuD,h1),MPT2B,PET(TuD,h2))
WednesdaysReportofLikely<-WPT1B<-PTE(TuPT1B,PET(WD,h1),TuPT2B,PET(WD,h2))
        WednesdaysReportofH2<-WPT2B<-PTE2(TuPT1B,PET(WD,h1),TuPT2B,PET(WD,h2))
ThursdaysReportofLikely<-ThPT1B<-PTE(WPT1B,PET(ThD,h1),WPT2B,PET(ThD,h2))
        ThursdaysReportofH2<-ThPT2B<-PTE2(WPT1B,PET(ThD,h1),WPT2B,PET(ThD,h2))
FridaysReportofLikely<-FPT1B<-PTE(ThPT1B,PET(FD,h1),ThPT2B,PET(FD,h2))
        FridaysReportofH2<-FPT2B<-PTE2(ThPT1B,PET(FD,h1),ThPT2B,PET(FD,h2))
SaturdaysReportofLikely<-SaPT1B<-PTE(FPT1B,PET(SaD,h1),FPT2B,PET(SaD,h2))
        SaturdaysReportofH2<-SaPT2B<-PTE2(FPT1B,PET(SaD,h1),FPT2B,PET(SaD,h2))
SundaysReportofLikely<-SuPT1B<-PTE(SaPT1B,PET(SuD,h1),SaPT2B,PET(SuD,h2))
        SundaysReportofH2<-SuPT2B<-PTE2(SaPT1B,PET(SuD,h1),SaPT2B,PET(SuD,h2))

WeeksBeliefs<-data.frame(Number=c(0,1,2,3,4,5,6,7),
Day=c( 'Initial', 'Monday', 'Tuesday', 'Wednesday', 'Thursday', 'Friday', 'Saturday',
'Sunday'),
BeliefStrength=c(initialPh1, MPT1B, TuPT1B, WPT1B, ThPT1B, FPT1B, SaPT1B,
SuPT1B))

WeeksBeliefs
```

```
##   Number       Day BeliefStrength
## 1      0   Initial      0.2500000
## 2      1    Monday      0.6089655
## 3      2   Tuesday      0.9310918
## 4      3 Wednesday      0.9954281
## 5      4  Thursday      0.9990179
## 6      5    Friday      0.9996094
## 7      6  Saturday      0.9999164
## 8      7    Sunday      0.9999383

plotdata(WeeksBeliefs, 'Number', 'BeliefStrength')
```



5 Salmon's Bridge to Kuhn

Thomas S. Kuhn's revolutionary *Structure of Scientific Revolutions* espoused an image of science which is unsure and somewhat more patchwork than the logical empiricists of the age fancied. Such opponents to Kuhn's view seemed to take his picture to the extreme, interpreting it to mean he thought scientist's irrational and subjective. One of Kuhn's large concepts outlined in his *Structure*, was the idea of a paradigm. With this paradigm came the necessity of competing theories in pre- and inter-paradigm periods, as well as the expulsion of the idea of continuous progression of science. This left theories, to some, to seem more like choices made on the whim of humans as opposed to intrinsic and true progress towards a real scientific description of reality. On the contrary, Kuhn seems not to think so little of science, but only hoped to portray a more accurate picture of what it is actually like than what it should ideally be. Though, critics persisted, pointing out that the competition of theories prevalent in his view damaged the more common view of a unified science always narrowing towards the truth.

Kuhn outlined five criteria for good scientific theories: accuracy, consistency, scope, simplicity, and fruitfulness. These criteria can be taken to be atleast a good portion of the criteria by which Kuhn would argue that scientists consider when deliberating on which theory is correct or most 'right' out of a stable of choices. Accuracy is obviously how well the theory predicts observed phenomena. Consistency is the overall agreement of the theory both with itself, and with other accepted theories of the context. Scope is how wide an applicability the theory has, as a theory that seems to hold true through a larger berth of reality is probably more likely to be true than one that applies only in a small number of cases. Fruitfulness, or the usefulness of the theory, is how much one can gain by accepting a certain theory. And lastly, simplicity, seemingly the most ambiguous of the criteria, is what seems to be a more subjective criteria that pertains to the beauty or compactness of a theory.

Wesley Salmon, as mentioned in the overview, is a strong supporter of Bayesianism and specifically, how we've defined it. As such, he claims to subscribe to a form of Bayesian Inference for the considerations on opposing theories and also takes probabilities to be potentially referential to belief values of individuals. So, when confronted with Kuhn's unique picture of science, he saw a region where Bayesian Inference was wholly applicable in Kuhn's picture, as he outlined in [4].

Salmon considers the ingredients of the modified Bayes' Theorem provided in section 3. The plausibility arguments, as brought up in section 2, where trained scientists can reason some theories to be more or less plausible, he sees as equivalent to the determination of the prior probabilities (of the form $P(T_x|B)$). Salmon claims these prior probabilities can be based on essentially three criteria, what he calls pragmatic, formal, and material. Pragmatic criteria are those pertaining to the context or credentials of the individual involved in the production of the theory. Formal criteria are those considerations of compatibility with already accepted truths. And finally, material criteria are those concerned with the internal workings of the theory, including the somewhat subjective ideas of simplicity or elegance.

Salmon's criteria of prior probabilities align nicely with Kuhn's criteria for good scientific theories. As Salmon states in [4], "Kuhn's criteria of consistency (broadly construed) and simplicity seem clearly to pertain to assessments of the prior probabilities of theories. They cry out for a Bayesian interpretation." Salmon takes this insight to form the conclusion that the plausibility arguments employed by scientists when comparing these theories is effectively the determination by them of the prior probabilities in Bayes' Theorem.

Overall, Salmon narrows down to the case of two competing theories and even offers a new mathematical relationship of Bayesian flavor to formalize the Kuhnian comparison of theories in a discipline in crisis. He claims the hard to pin probability of theories not yet thought of to be avoidable by creating a relative likelihood of being true, as shown below.

$$\frac{P(T_1|E.B)}{P(T_2|E.B)} = \frac{P(T_1|B) * P(E|T_1.B)}{P(T_2|B) * P(E|T_2.B)}$$

This clearly follows from the modified Bayes' Theorem shown before. As both denominators of the probabilities given new evidence for both T_1 and T_2 are the same, by dividing the equations, the outcome relative equation circumvents any mention of theories outside the scope of consideration. This equation for comparison is called by Salmon the *Bayesian algorithm for theory preference*.

6 Conclusion

Thus, the picture of science as painted by Kuhn left a gap in which scientists had the freedom to choose between different competing theories. This gap was taken by some to be a clear indication that Kuhn made out science to be irrational, with no real model for how the determination of these theories was carried out. Salmon, in his Bayesian school of thought, saw fit to apply his Bayesian ideas to this gap in determination. He shows that the criteria he outlined for the determination of prior probabilities aligned nicely with the criteria described by Kuhn when asked how theories are chosen by scientists. Salmon even brings the mathematical rigor of Bayesian Inference to this conclusion, showing that one can utilize the *Bayesian algorithm for theory preference* to decide between theories without even having to assign values to theories outside the scope of consideration.

References

[1] Internet Encyclopedia of Philosophy. *Theories of Explanation*. https://www.iep.utm.edu/explanat/. (Accessed on 04/28/2019).

[2] Stanford Encyclopedia of Philosophy. *Bayes Theorem*. https://plato.stanford.edu/entries/bayes-theorem/. (Accessed on 04/28/2019).

[3] Stanford Encyclopedia of Philosophy. *Wesley Salmon*. https://plato.stanford.edu/entries/wesley-salmon/. (Accessed on 04/28/2019).

[4] Wesley Salmon. Rationality and objectivity in science or tom kuhn meets tom bayes. *Scientific theories*, 14:175–204, 1990.