Qudit Hypergraphs and Function Encoding

Alexander J. Heilman

Lebanon Valley College

Moravian College
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Why Quantum Computers?

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- Possibly solve new problems

Information Processing

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Further, qu**d**its can be in a superposition of d different states:

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + \ldots + \alpha_{d-1} |d-1\rangle$$

Information Processing (cont.)

The collective system of physically close states is described by the tensor-product of their states. A two qubit state would look something like:

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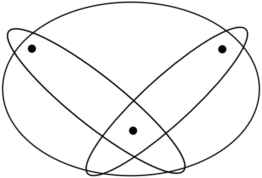
One class of quantum states that has proven useful for quantum computation are hypergraph states

Hypergraph States

Hypergraphs are a combination of vertices (dots) and edges (circles).



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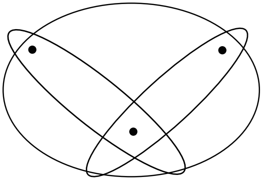

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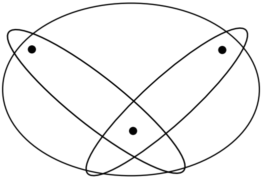

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$$|\psi\rangle = |000\rangle + |001\rangle + |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle - |111\rangle$$

Function Encoding

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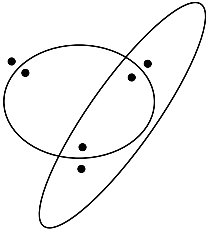

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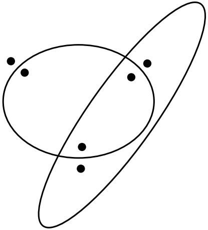

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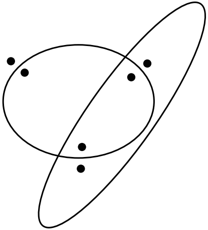
$$= |000\rangle + |001\rangle + |002\rangle + |010\rangle + |011\rangle + |012\rangle + |020\rangle + |021\rangle + |022\rangle + |100\rangle + \omega |101\rangle + \omega |102\rangle + |110\rangle + \omega^2 |111\rangle + |112\rangle + |120\rangle + |121\rangle + \omega^2 |122\rangle + |200\rangle + \omega |201\rangle + \omega |202\rangle + |210\rangle + |211\rangle + \omega^2 |212\rangle + |220\rangle + \omega^2 |221\rangle + |222\rangle$$

Function Encoding (cont.)

Alternatively, these states can be described with uniquely encoded functions (mod d).



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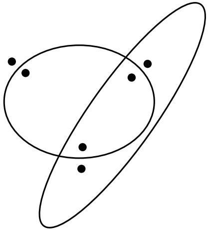

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The function, f, determines the phase for each individual element of the superposition. For example:

$$f(1, 2, 2) = 1 \cdot 2 \cdot 2 + 1^2 \cdot 2^2 = 2 \longrightarrow \omega^2 |122\rangle$$

Encoding Process

The general map for encoding a hypergraph as a function is as follows:

$$\overset{state}{|\psi\rangle} \underset{log\omega}{\longrightarrow} \overset{c-vec}{|c\rangle} \underset{S^{-1}matrix}{\longrightarrow} \overset{a-vec}{|a\rangle}$$

Where the a vector ($|a\rangle$) describes the coefficients of the state's function. Returning to the state is similar,

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The step from $|c\rangle$ to $|\psi\rangle$ is an entry-wise operation on the elements of a vector, while step $|a\rangle$ to $|c\rangle$ is achieved by multiplication of the S matrix

Encoding Process (cont.)

For example:

$$|\psi\rangle = |00\rangle + |01\rangle + |02\rangle + |10\rangle + \omega\,|11\rangle + \omega^2\,|12\rangle + |20\rangle + \omega^2\,|21\rangle + \omega\,|22\rangle$$

$$\downarrow$$

$$|\psi\rangle = \begin{bmatrix}1 & 1 & 1 & 1 & \omega & \omega^2 & 1 & \omega^2 & \omega\end{bmatrix}$$

$$\downarrow log_\omega$$

$$|c\rangle = \begin{bmatrix}0 & 0 & 0 & 0 & 1 & 2 & 0 & 2 & 1\end{bmatrix}$$

$$\downarrow S^{-1} Matrix$$

$$|a\rangle = \begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}$$

$x^0y^0 \mid x^0y^1 \mid x^0y^2 \mid x^1y^0 \mid x^1y^1 \mid x^1y^2 \mid x^2y^0 \mid x^2y^1 \mid x^2y^2$

$$f = xy$$

S Matrix

The S matrix has the form of a mod d Vandermonde matrix:

$$S_d = \begin{bmatrix} 0^0 & 0^1 & 0^2 & \cdots & 0^{d-1} \\ 1^0 & 1^1 & 1^2 & \cdots & 1^{d-1} \\ 2^0 & 2^1 & 2^2 & \cdots & 2^{d-1} \\ 3^0 & 3^1 & 3^2 & \cdots & 3^{d-1} \\ \vdots & \vdots & \vdots & \ddots & \\ (d-1)^0 & (d-1)^1 & (d-1)^2 & & (d-1)^{d-1} \end{bmatrix} \mod d$$

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For example, for d=5

$$S_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 & 1 \\ 1 & 3 & 4 & 2 & 1 \\ 1 & 4 & 1 & 4 & 1 \end{bmatrix} \bmod 5$$

Generalized Local Pauli Matrices

The Paulis are a basis for $d \times d$ operators (actions) on individual qudits. They can be used to describe all local actions on a state. For qubits ($d = 2$), they are standard:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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We've considered one possible generalization of the Paulis for local d dimensional actions, the Ps (generalized X) and Qs (generalized Z). For example:

$$P_{01} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} Q_0 = \begin{bmatrix} \omega & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Generalized Local Pauli Matrices (cont.)

The Ps act as subspace permutations and the Qs act as local phase shifts on the state vector ($|\psi\rangle$).

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Pauli operators are important for error-correcting applications.

GLP Actions on Encoded Functions

In order to see the effects of the generalized Paulis on encoded functions, we must consider the map of encoding,

$$|a'\rangle \leftarrow S^{-1} \leftarrow log_{\omega} \leftarrow |\psi\rangle \leftarrow exp_{\omega} \leftarrow S \leftarrow |a\rangle$$

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$$f(x,y) \equiv 0,\ xy,\ xy^2,\ x^2y,\ xy^2 + x^2y,\ x^2y^2,$$
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All states n=2, d=3 can be transformed into one of the above functions with the GLP

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Stabilizers are important in application and theory.

GLP Stabilizers (cont.)

One family of stabilizers found to work for any d or number of qudits is the 'Virginia Reel'

$$P_{VR} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

GLP Stabilizers (cont.)

One family of stabilizers found to work for any d or number of qudits is the 'Virginia Reel'

$$\mathrm{P}_{VR} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

And its transpose,

$$\mathrm{P}_{VR}^{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

GLP Stabilizers (cont.)

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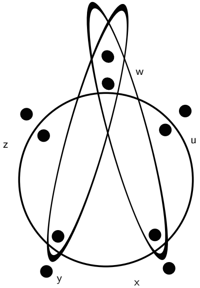
And its transpose,

$$\mathrm{P}_{VR}^{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

These will stabilize any function of the form f=(x+y)·(other variables)

GLP Stabilizers (cont.)



$$f(x, y, z, w, u) = (x + y)(zu + w^2)$$

The Virginia Reel is exponentiated from a hermitian matrix (M) and a certain constant $2\pi i/d$ ($P_{VR}=e^{(2\pi i/d)M}$). Its transpose/inverse is exponentiated from the same hermitian matrix and the negative of the same constant ($P_{VR}^T=e^{(-2\pi i/d)M}$). However, for any real t, $e^{itM} \otimes e^{-itM}$ is also a stabilizer for functions of the form f=(x+y)·(other variables). Thus, there is an infinite family of stabilizers for such functions.

Thank You!

Lebanon Valley College Mathematical Physics Research Group
http://quantum.lvc.edu/mathphys/