

Quantum Algorithms & Qiskit

Alexander J. Heilman

August 19, 2021

Intentions

Introduce you to some basic quantum algorithms

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First, the QFT

The Quantum Fourier Transform

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The Quantum Fourier Transform

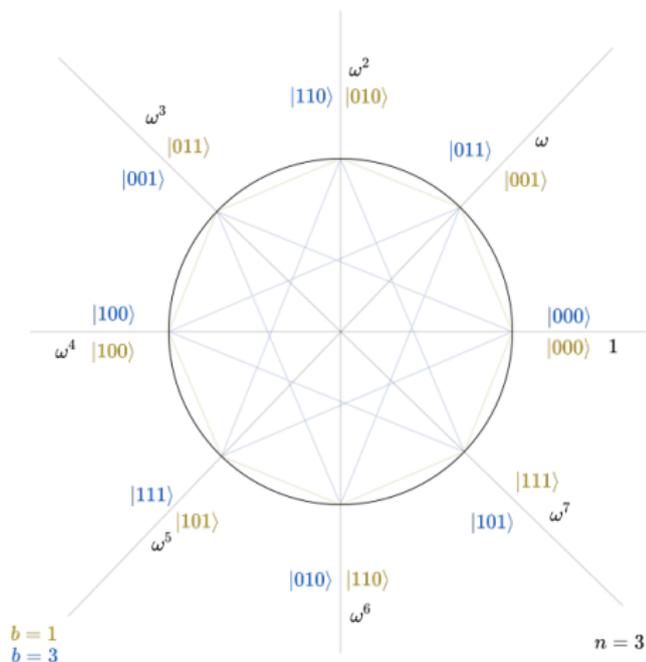
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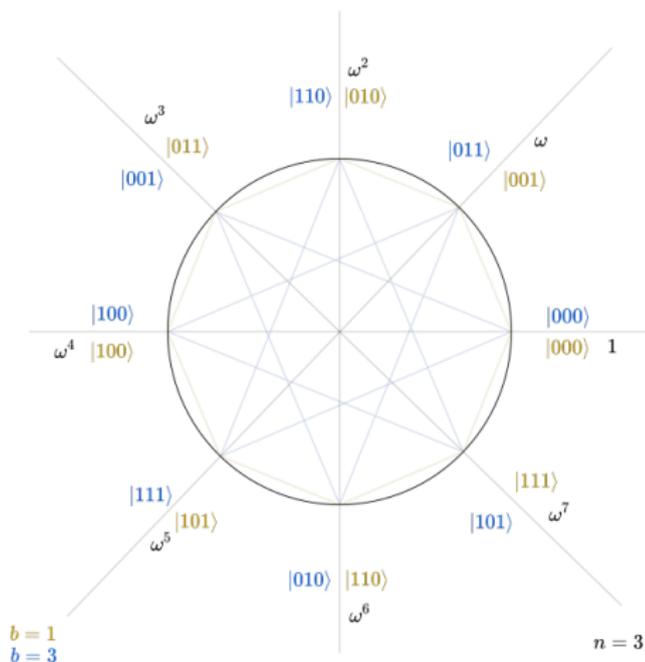
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But how?

QFT: The gory details

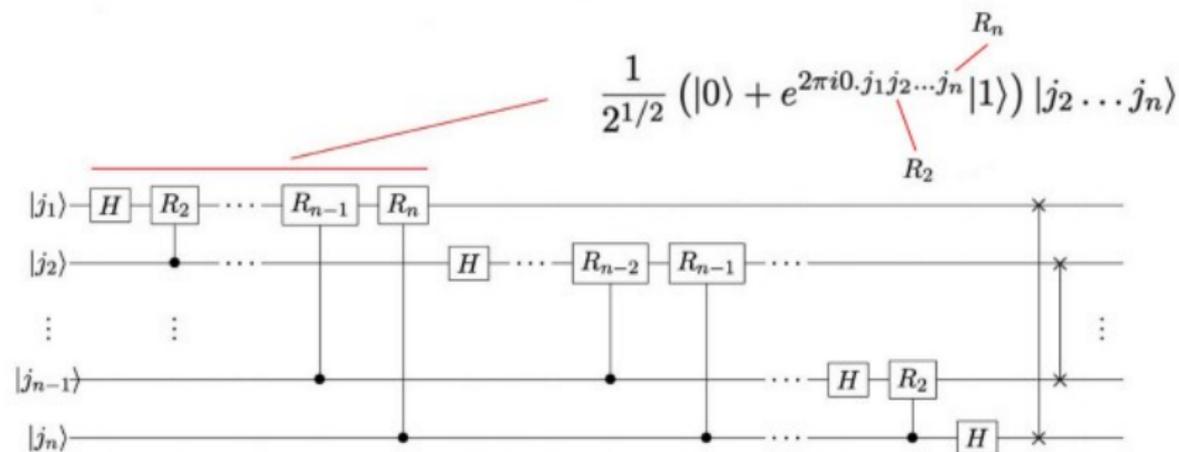
The explicit action of the n-qubit QFT on some given basis vector is

$$|j_1, \dots, j_n\rangle \longrightarrow \frac{(|0\rangle + e^{2\pi i 0 \cdot j_n} |1\rangle) \otimes (|0\rangle + e^{2\pi i 0 \cdot j_{n-1} j_n} |1\rangle) \otimes \dots \otimes (|0\rangle + e^{2\pi i 0 \cdot j_1 \dots j_n} |1\rangle)}{2^{n/2}}.$$

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It's easier to see in matrix form

QFT: The gory details II

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Not so interesting yet, but it's usefulness will soon be found in phase estimation

Phase Estimation

Phase Estimation takes some given unitary operator and an associated eigenvector and returns the corresponding eigenvalue.

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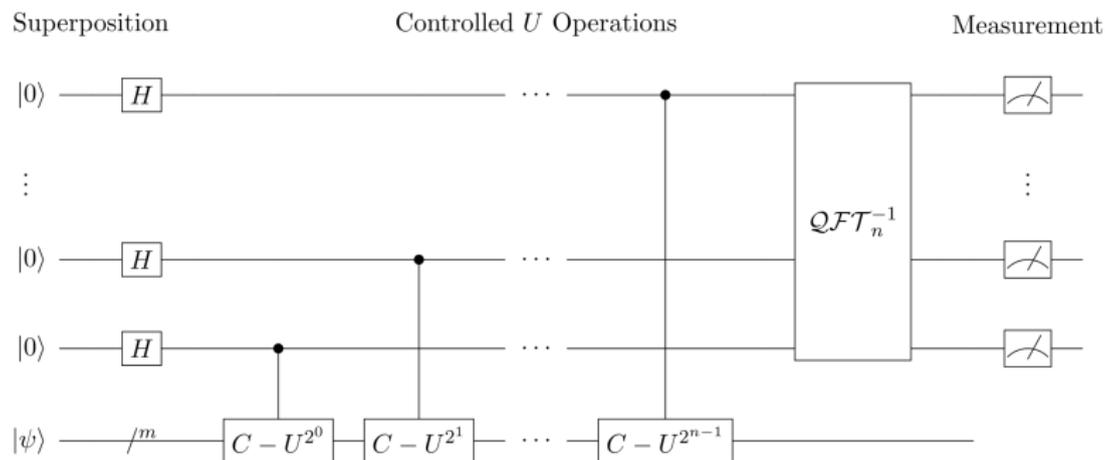
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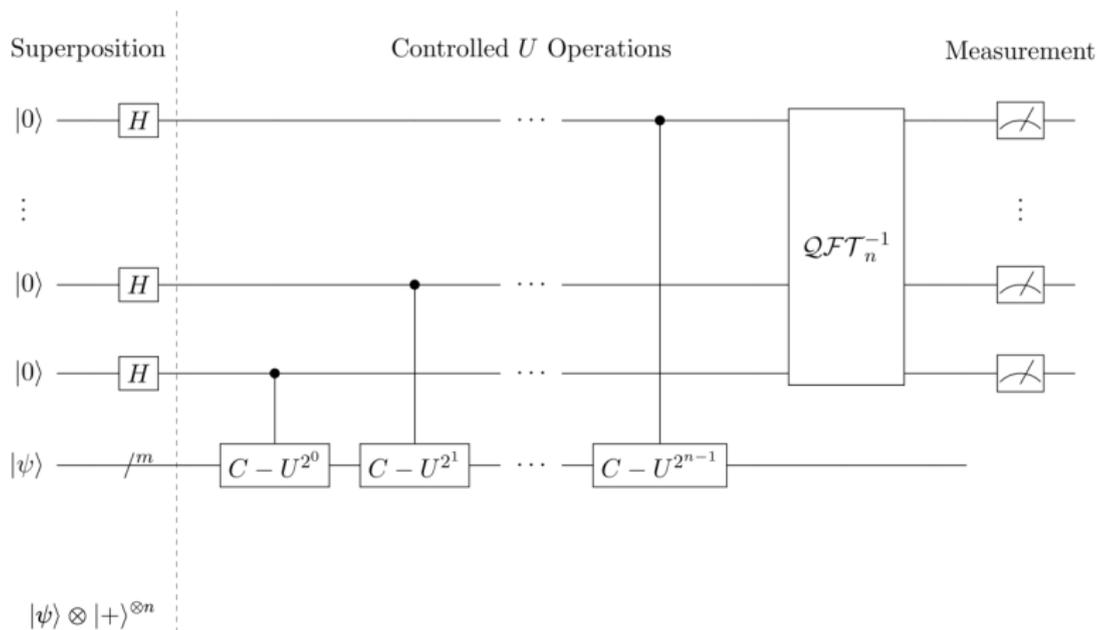
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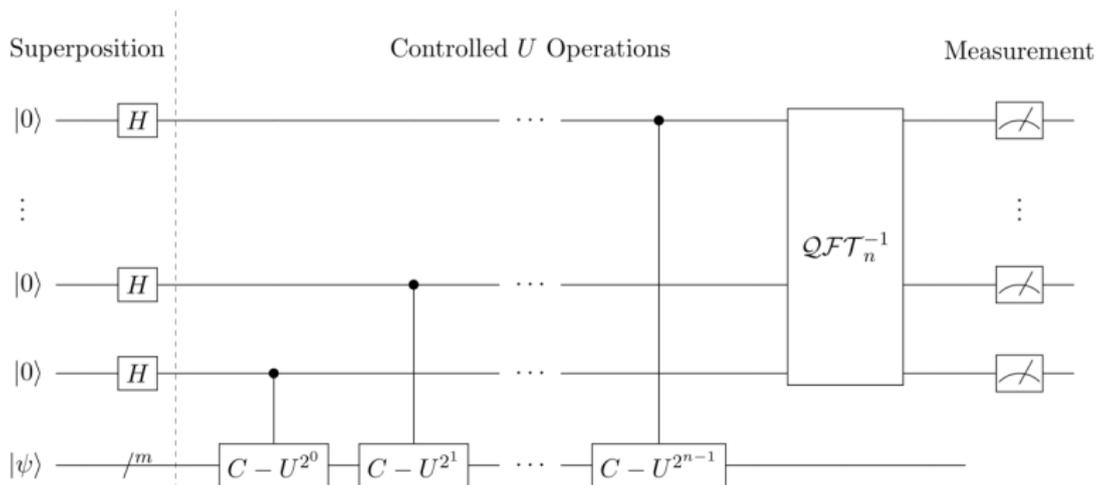
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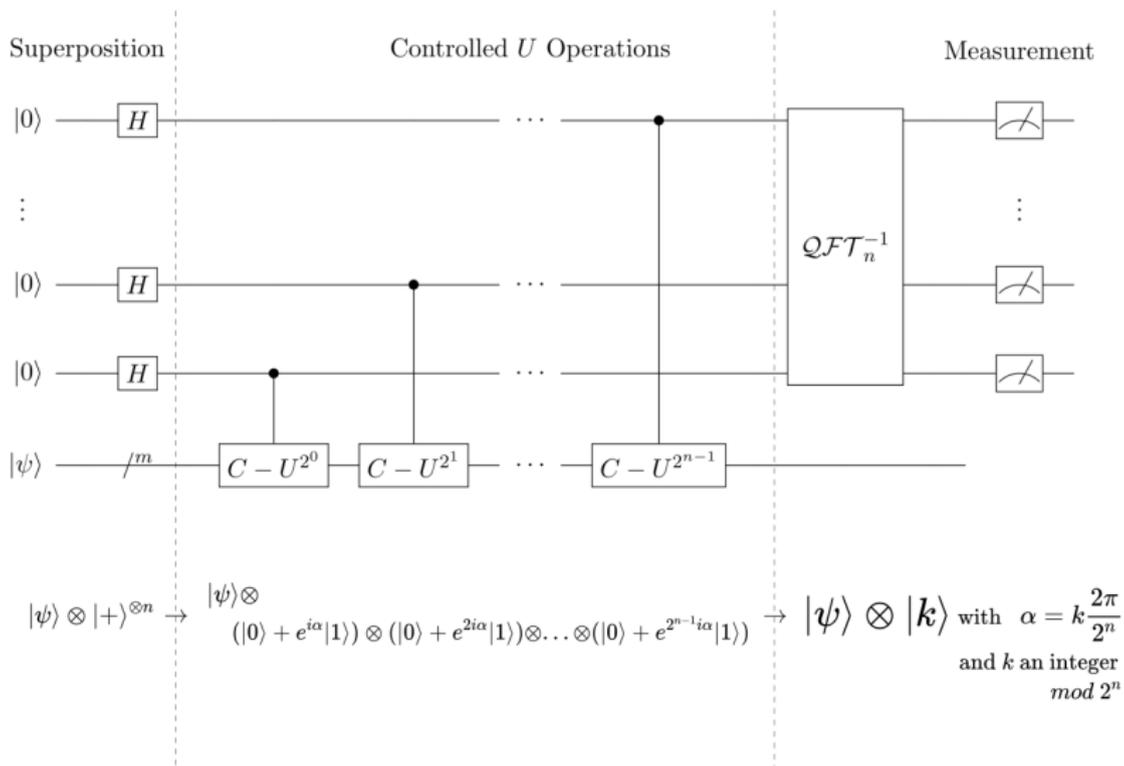


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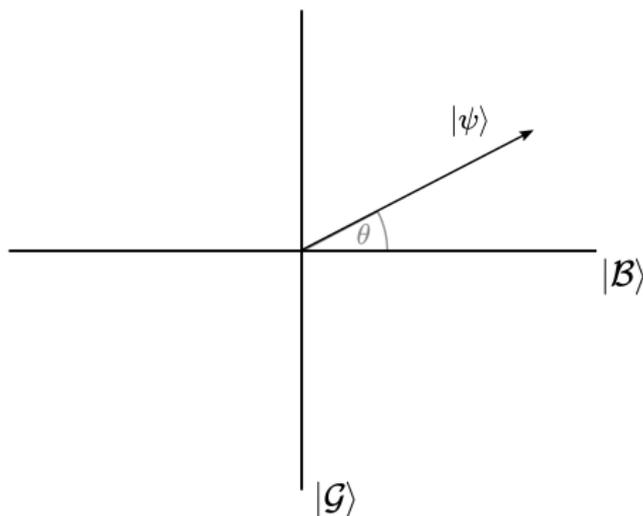
$$|\psi\rangle \otimes |+\rangle^{\otimes n} \rightarrow |\psi\rangle \otimes (|0\rangle + e^{i\alpha}|1\rangle) \otimes (|0\rangle + e^{2i\alpha}|1\rangle) \otimes \dots \otimes (|0\rangle + e^{2^{n-1}i\alpha}|1\rangle)$$

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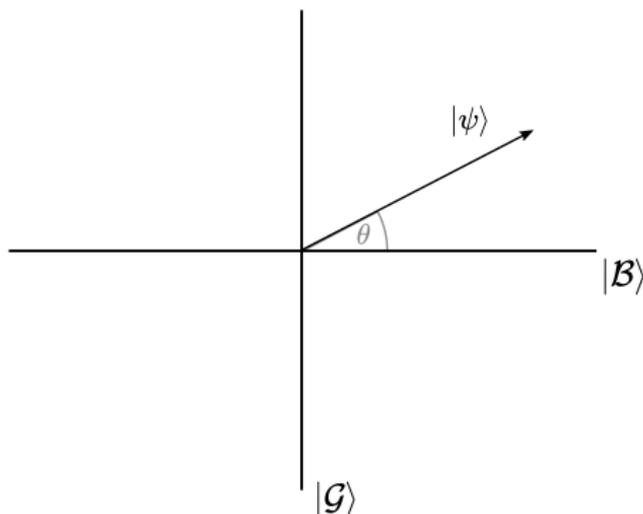
Amplitude Amplification

Amplitude estimation takes some statevector partitioned into a good and a bad subspace



Amplitude Amplification

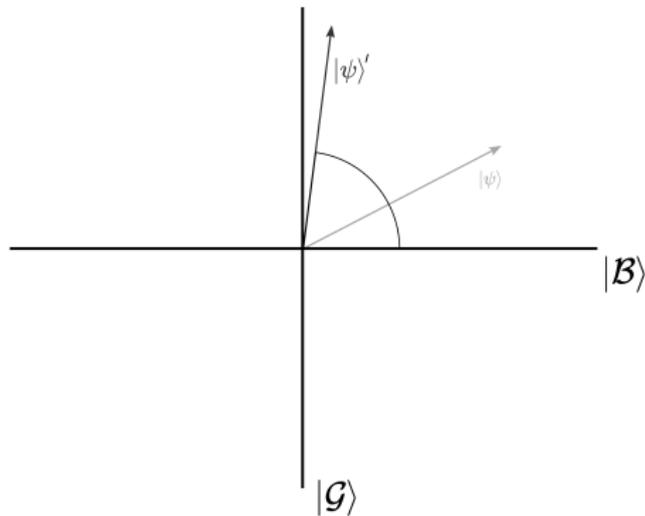
Amplitude estimation takes some statevector partitioned into a good and a bad subspace



$$|\psi\rangle = \sin(\theta)|\mathcal{G}\rangle + \cos(\theta)|\mathcal{B}\rangle.$$

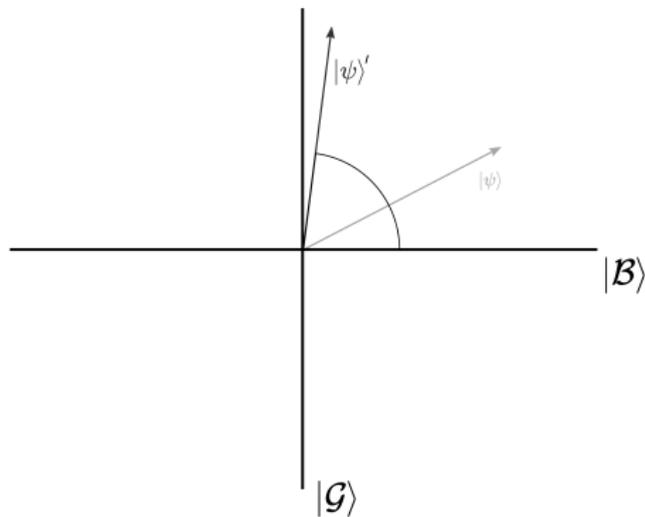
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This is accomplished with consecutive applications of a special operator

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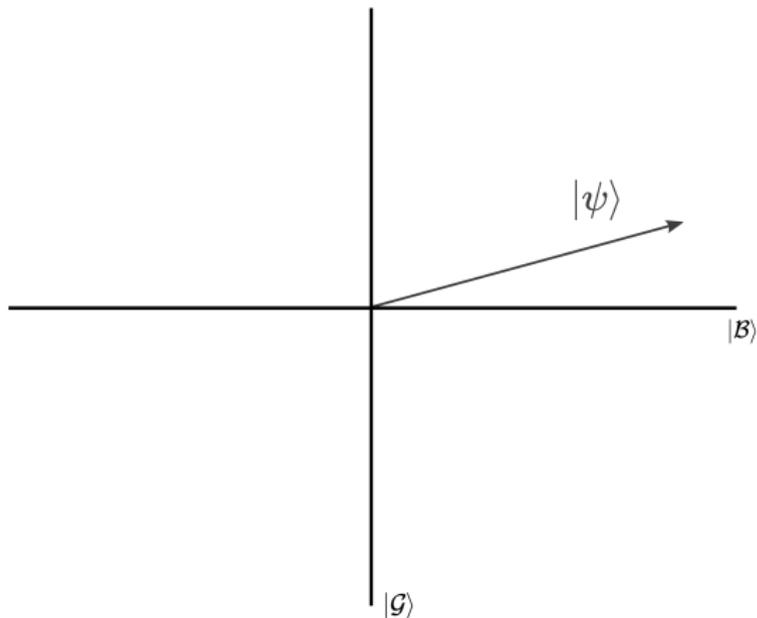
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Let's see what they do!

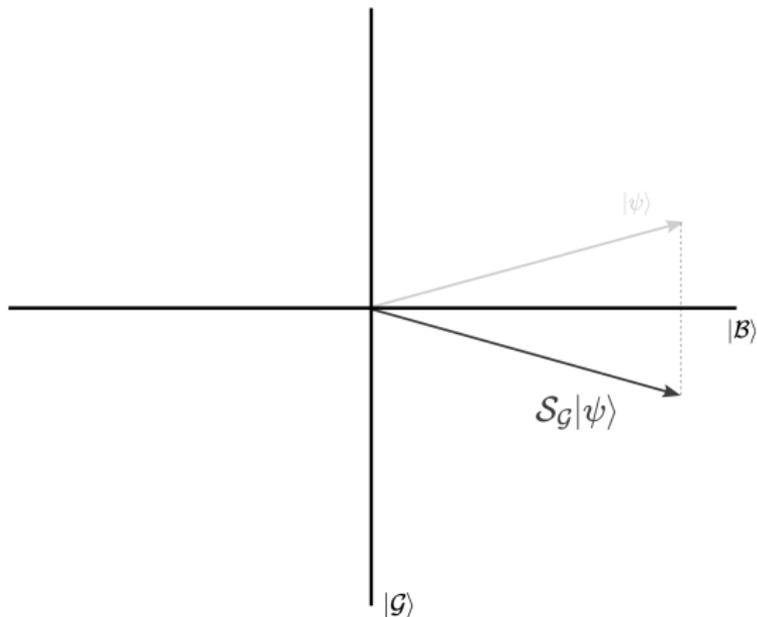
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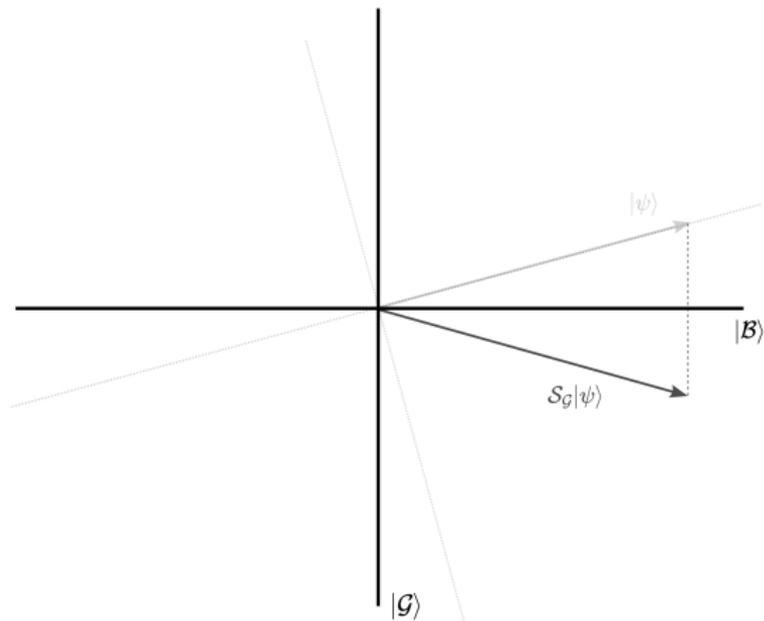
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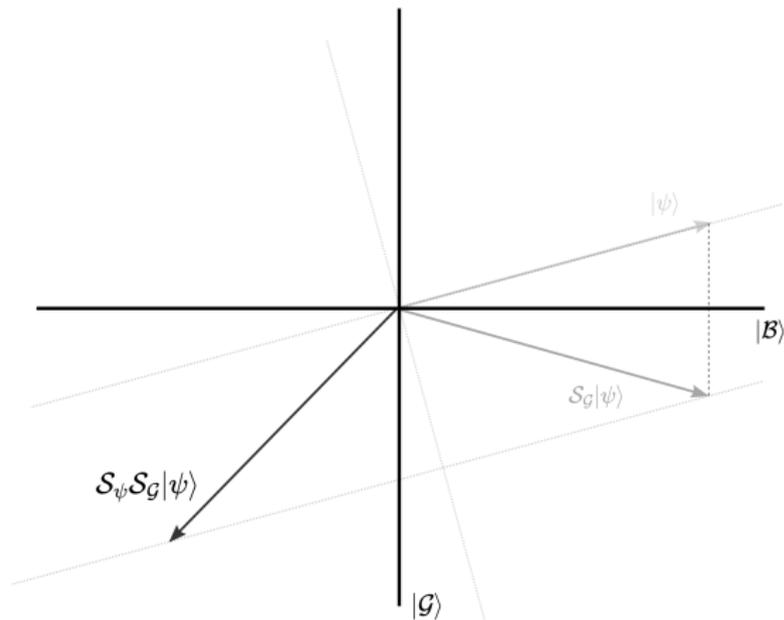
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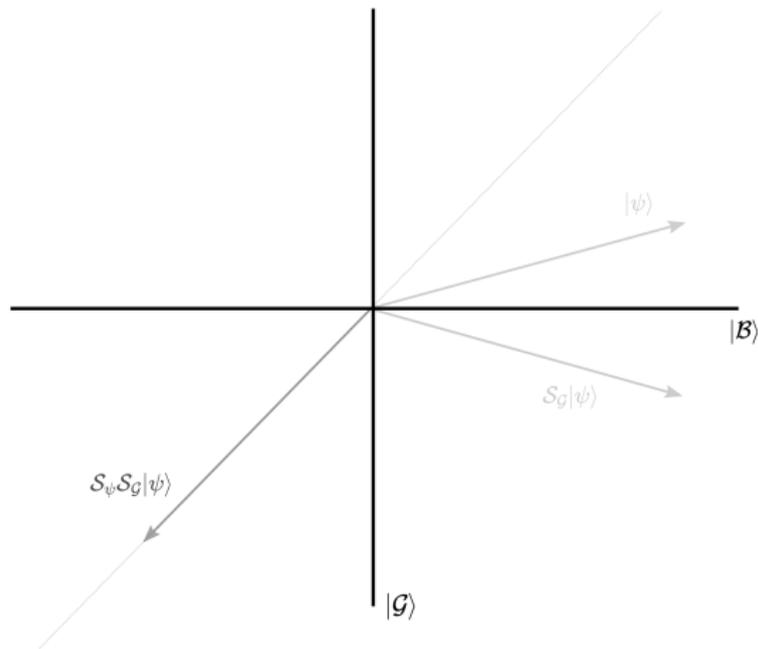
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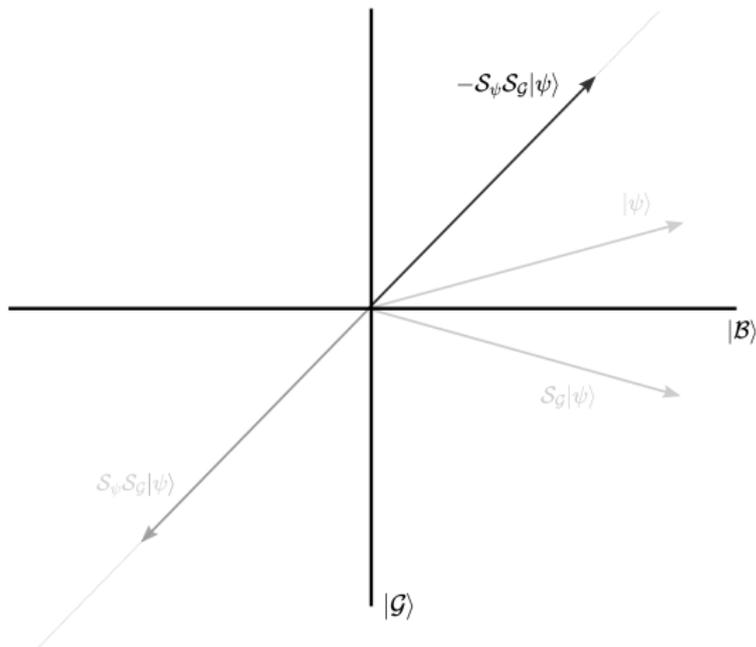
Amplitude Amplification: Visualization III

Now, negate the resulting statevector



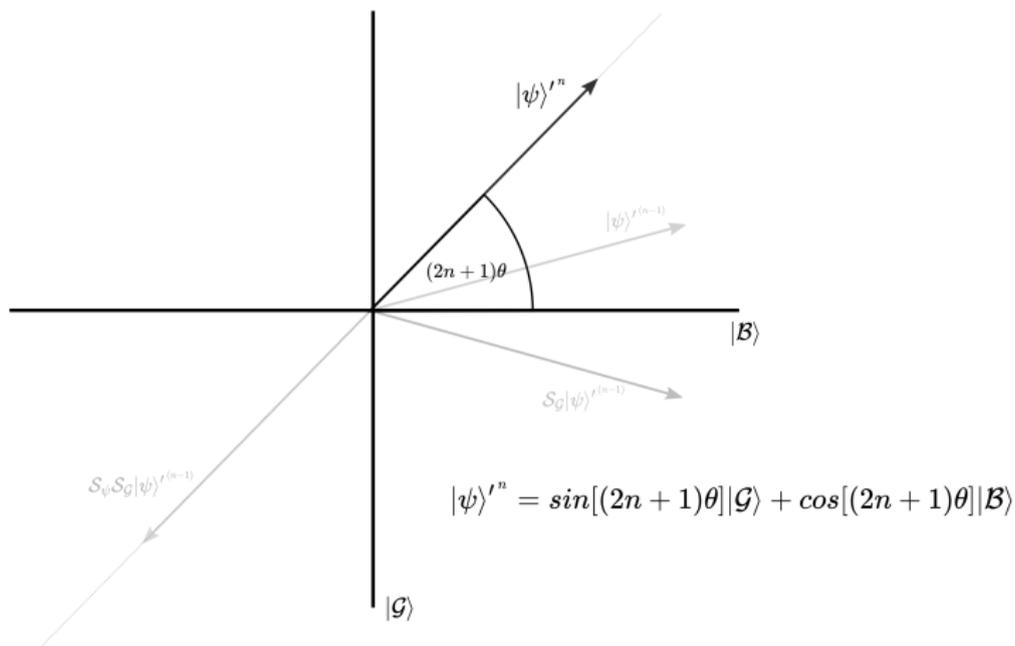
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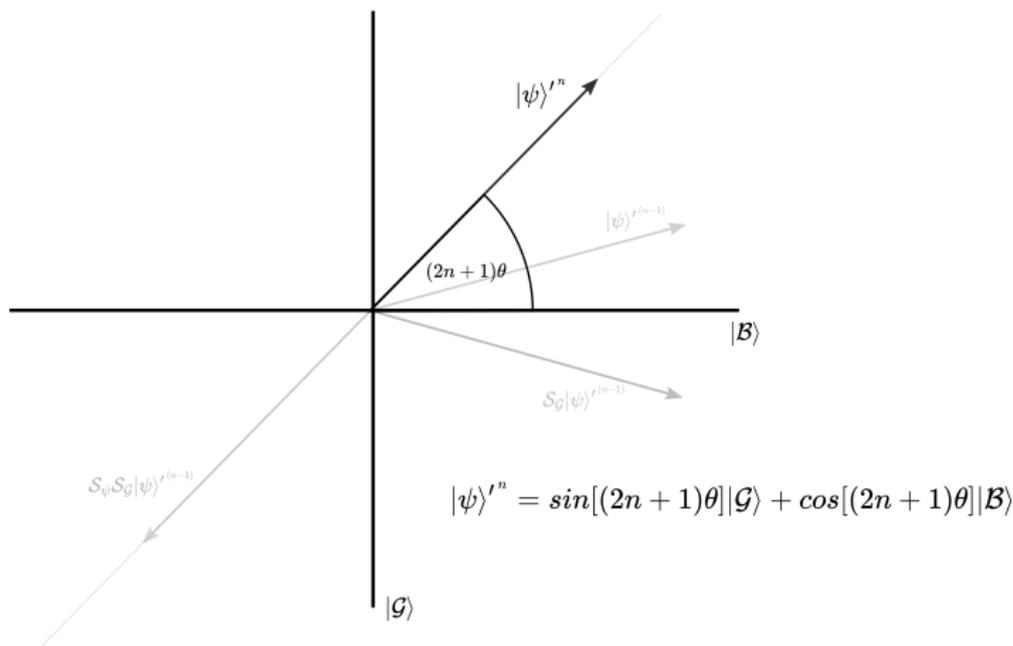
Amplitude Amplification: Visualization IV

And voilà!



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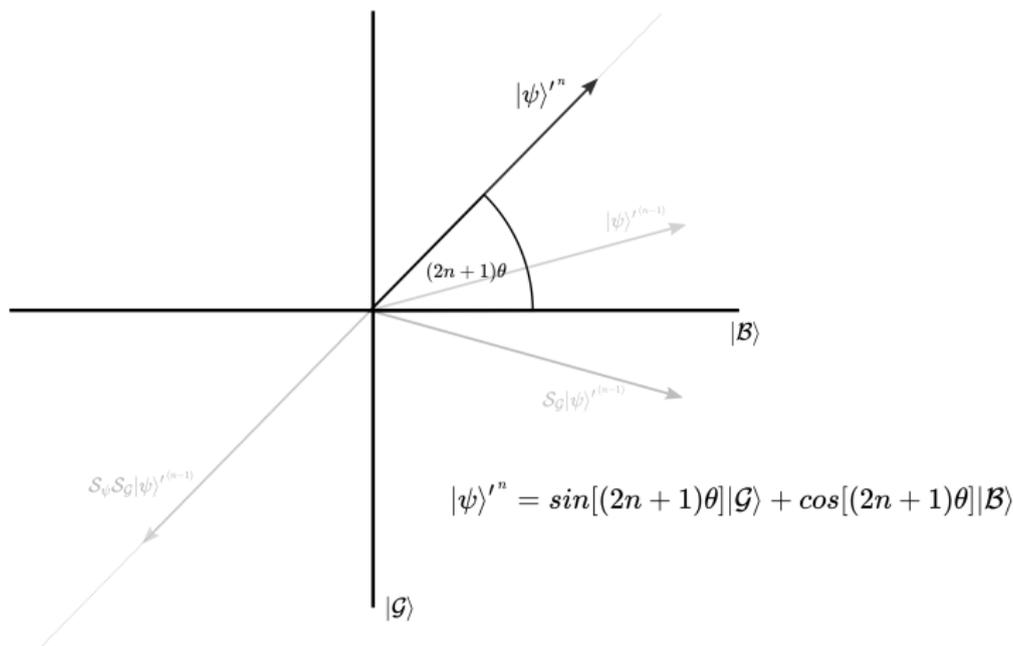
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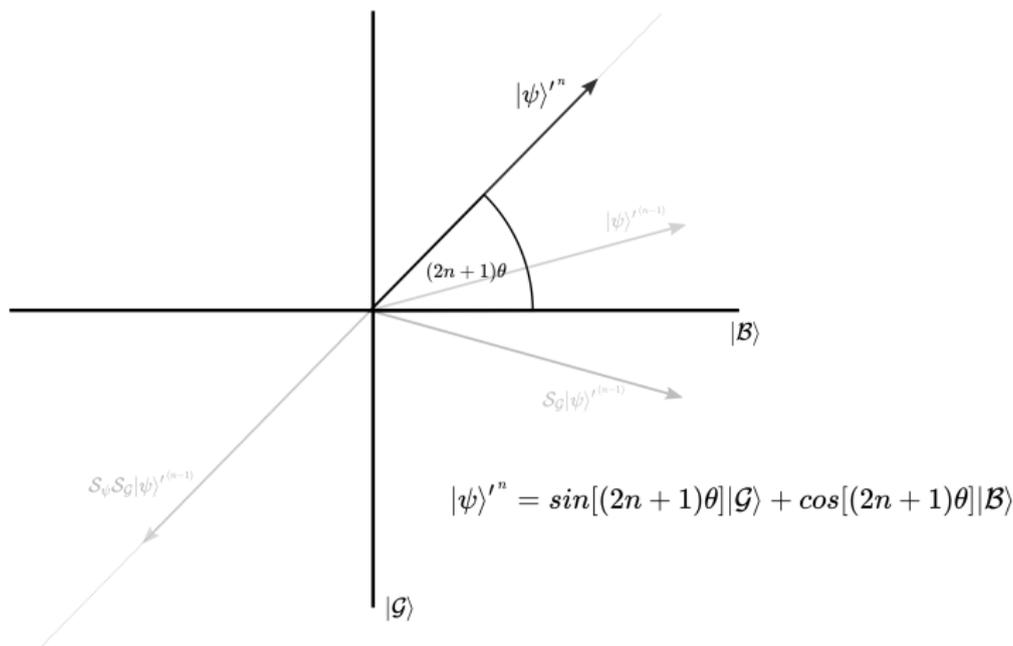
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Now, let's do some coding!

Qiskit

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Running the QFT

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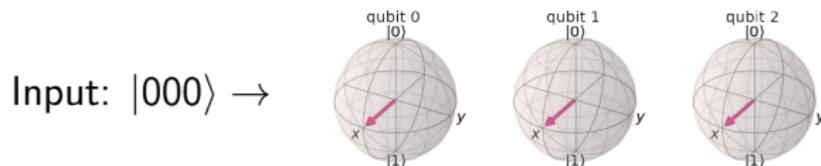
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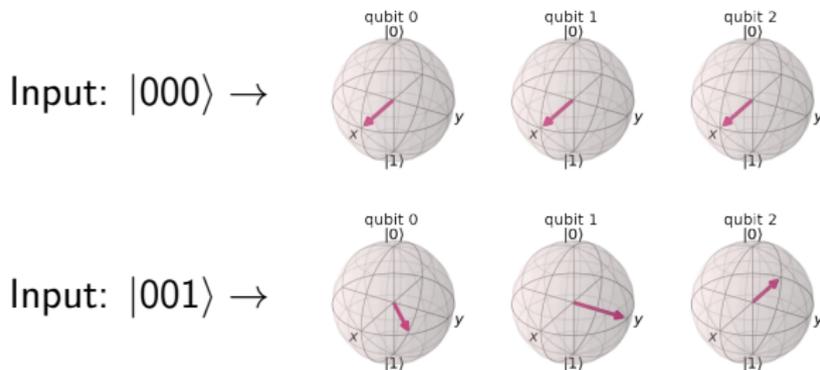
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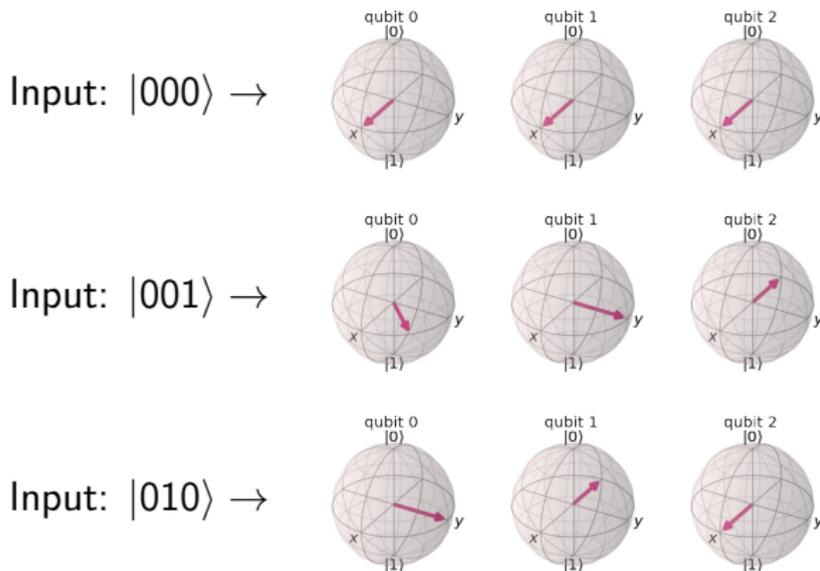
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Moving forward, I'll just link the code online.

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We already know that $|1\rangle$ is an eigenvector of basic phase gates.

Finding Phases: $CP(\pi/4)$

This time, we'll use the qasm simulator

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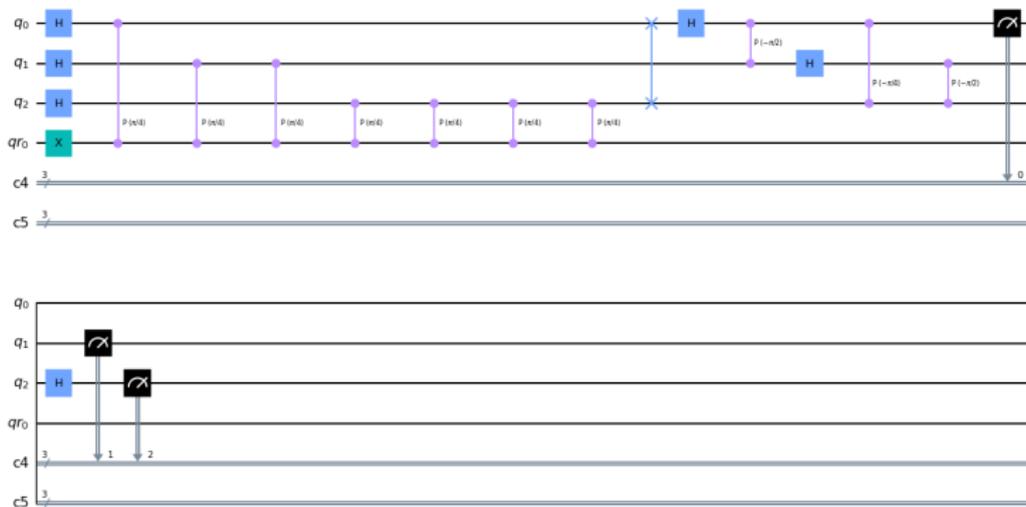
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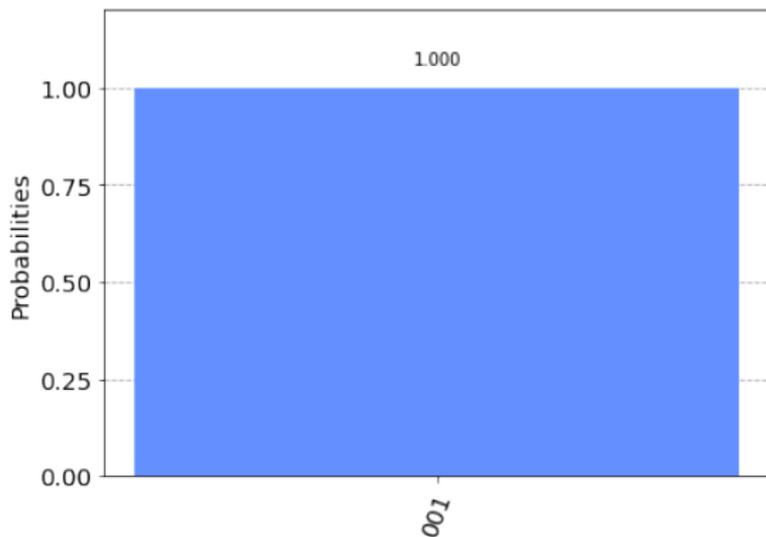


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Running the simulation gives a simulated measurement set:

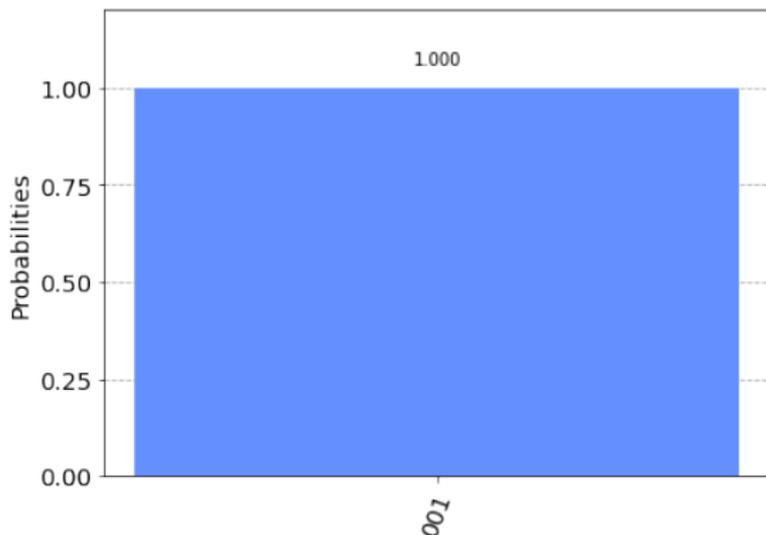
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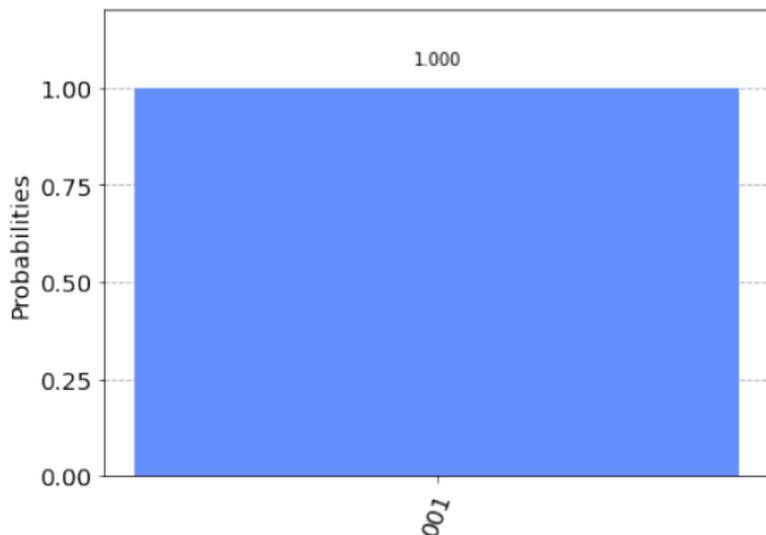
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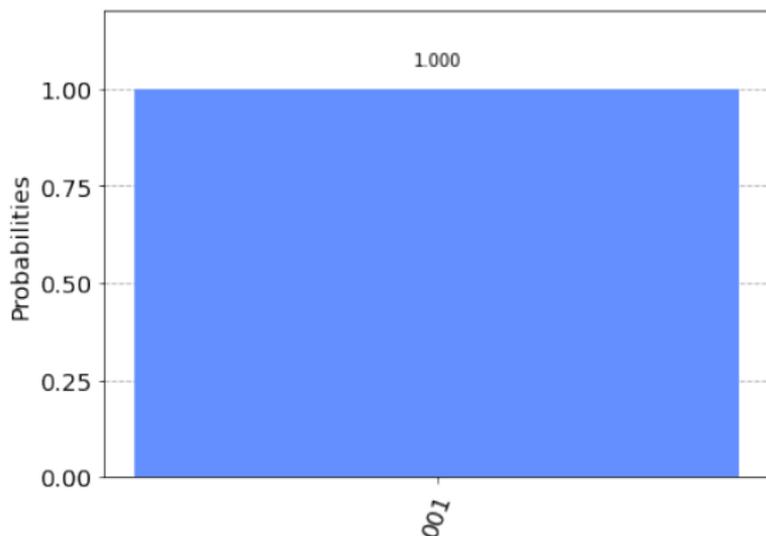
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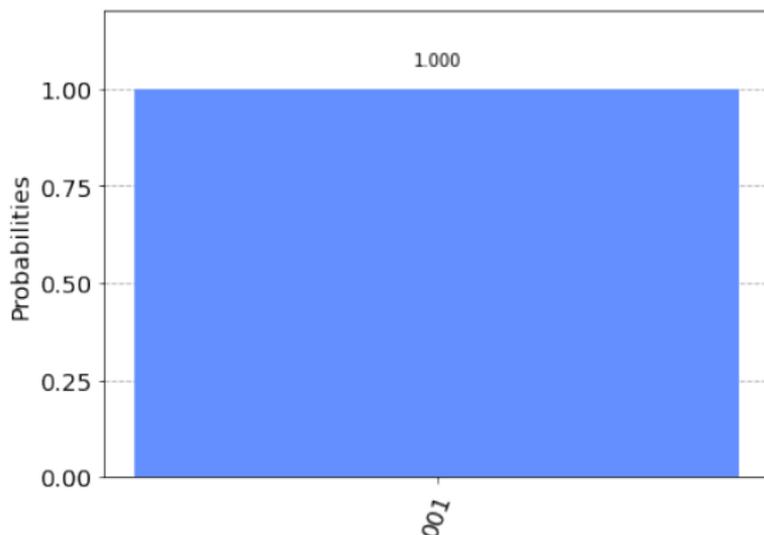


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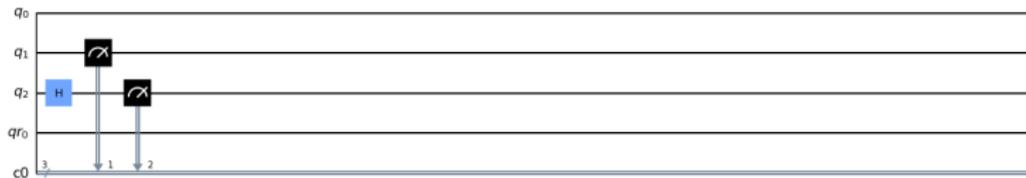
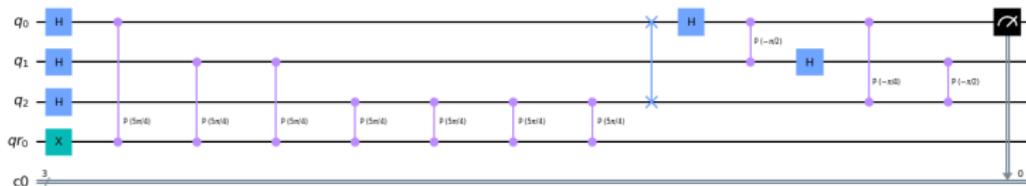
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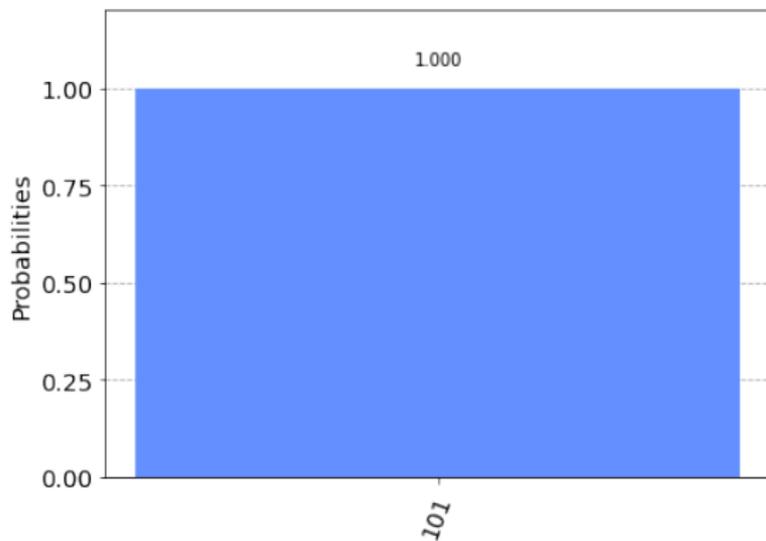


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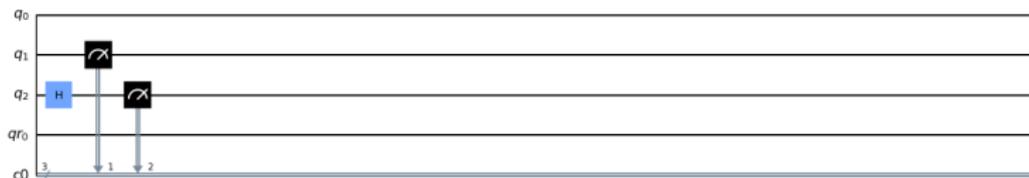
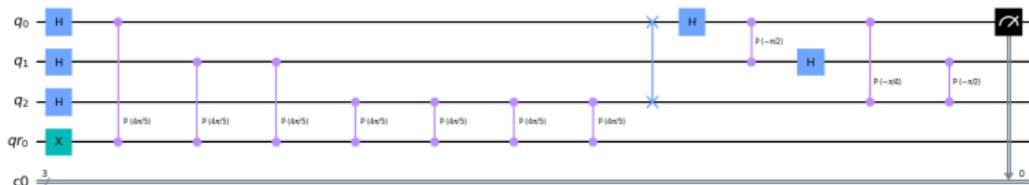
Yup!

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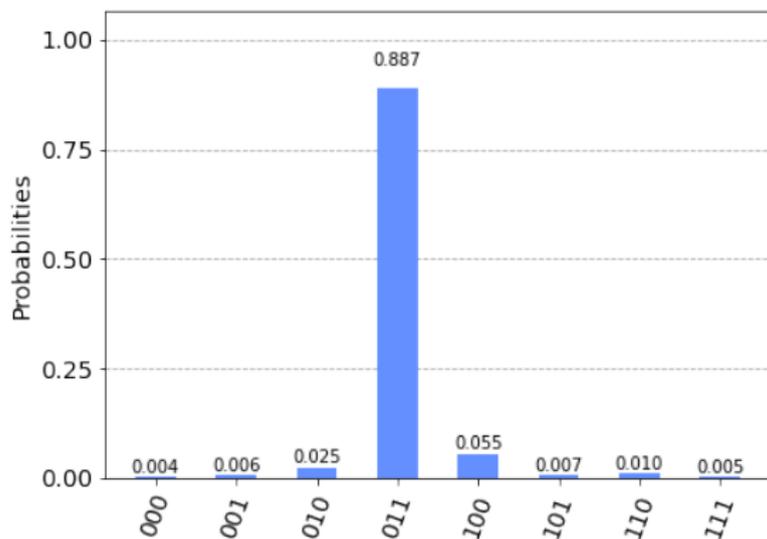


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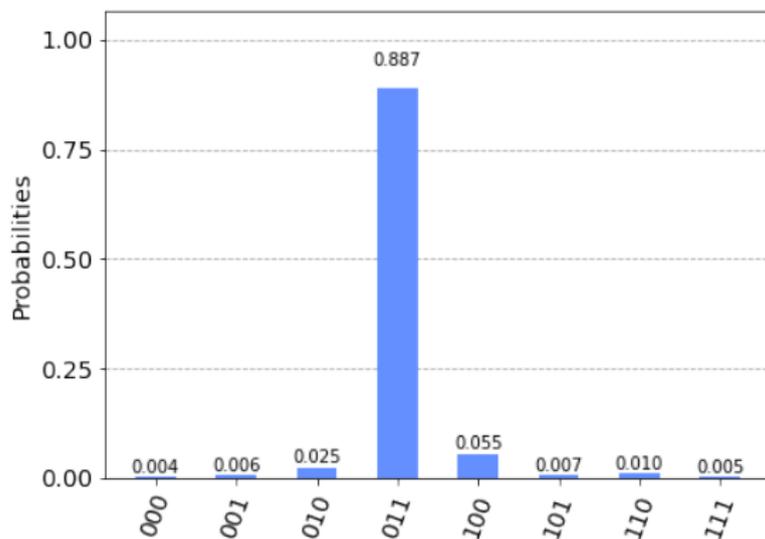
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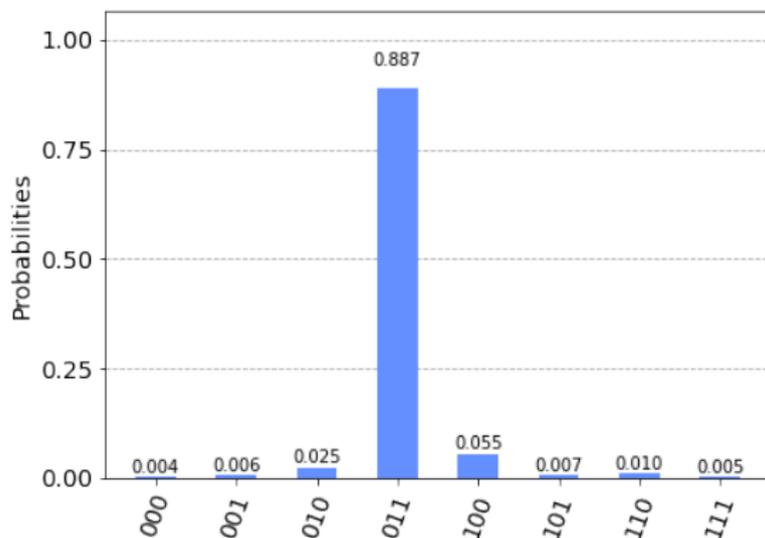
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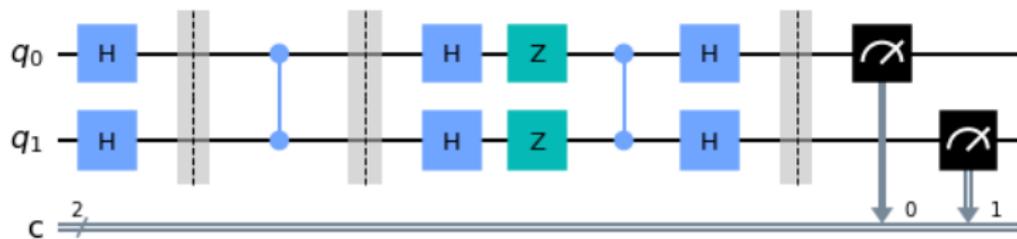


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Let's do some amplitude amplification!

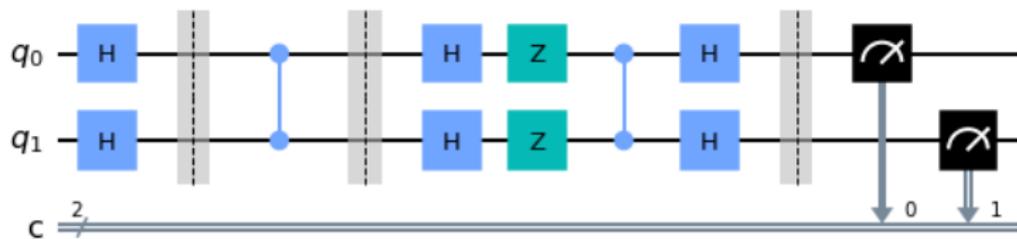
Amplitude Amplification in Practice

Below is a simple 2-qubit circuit for amplitude amplification that searches for the $|11\rangle$ state:



Amplitude Amplification in Practice

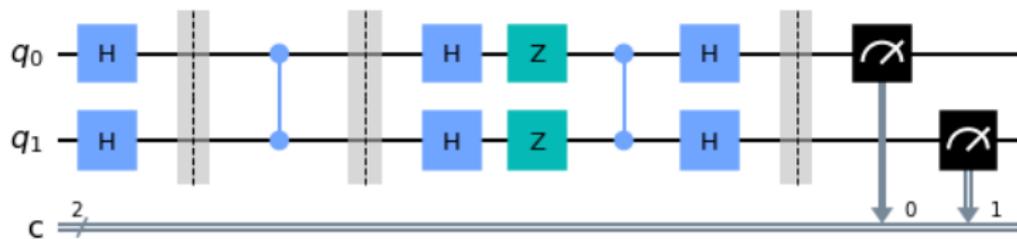
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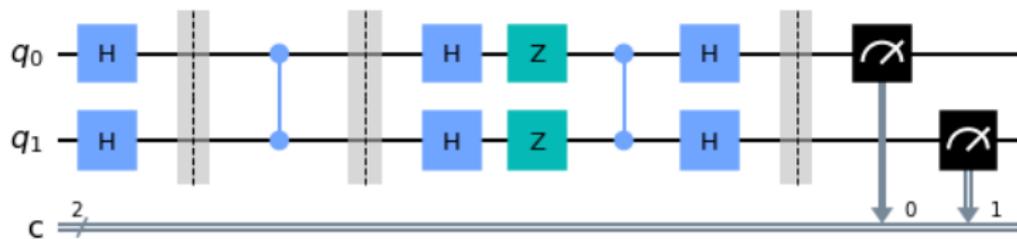


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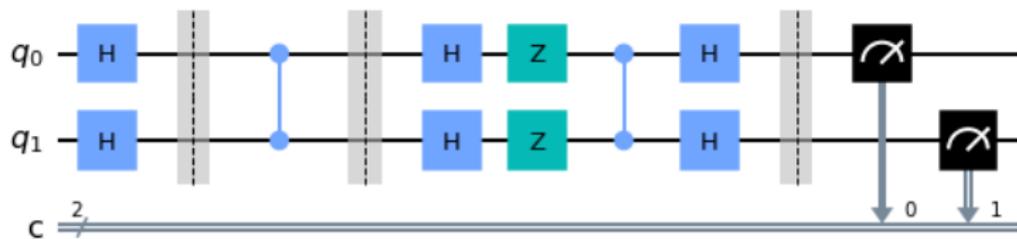
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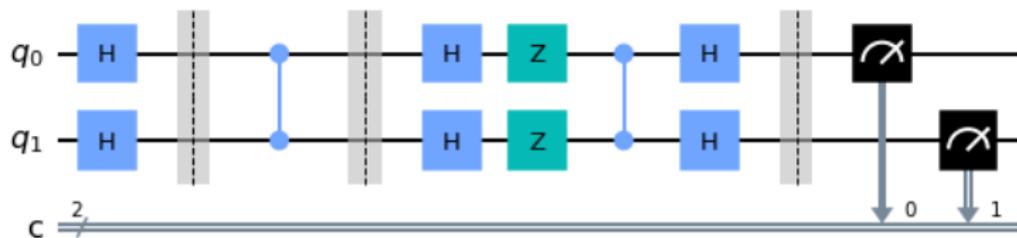
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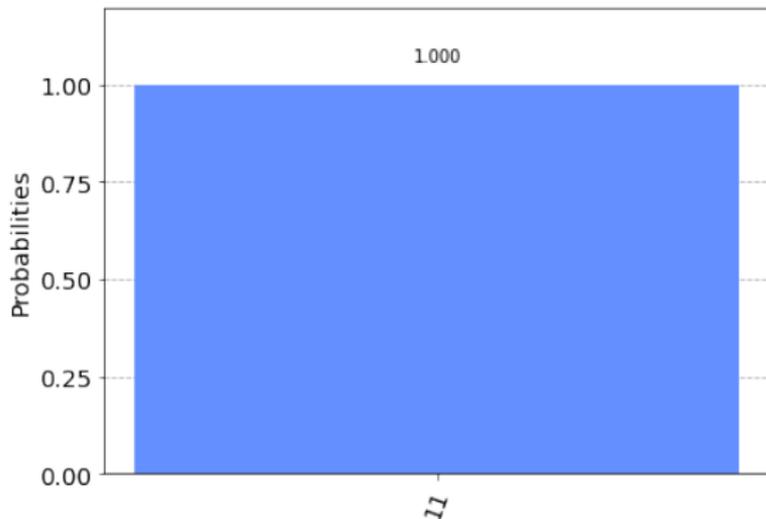
The third section flips the state about the original vector.

We only need to run the operations once as $n = 1$ here.

Let's run it!

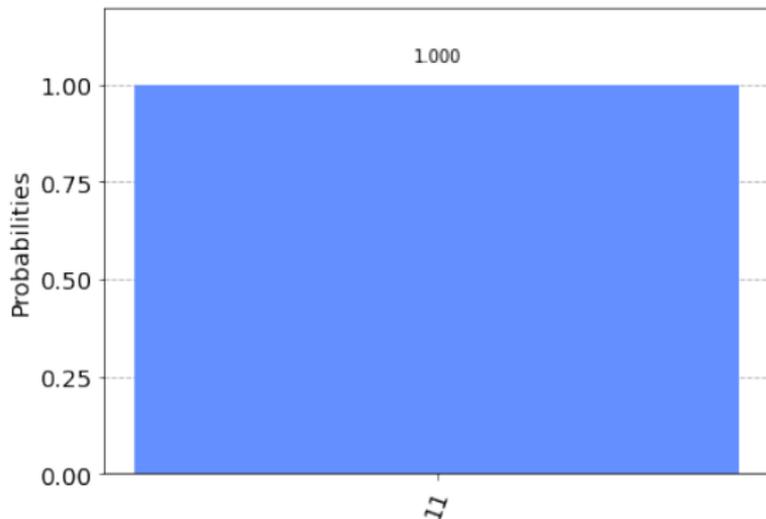
Amplitude Amplification in Practice: Simulation

Running the previous circuit on the simulation we used for phase estimation:



Amplitude Amplification in Practice: Simulation

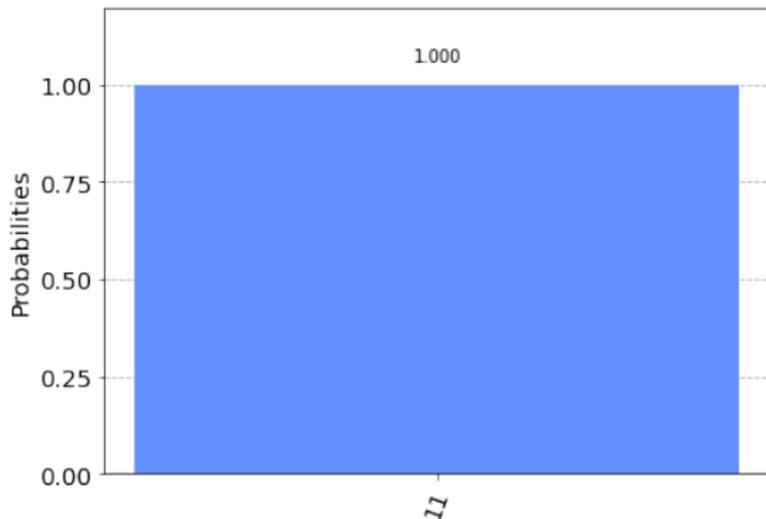
Running the previous circuit on the simulation we used for phase estimation:



Perfect results!

Amplitude Amplification in Practice: Simulation

Running the previous circuit on the simulation we used for phase estimation:



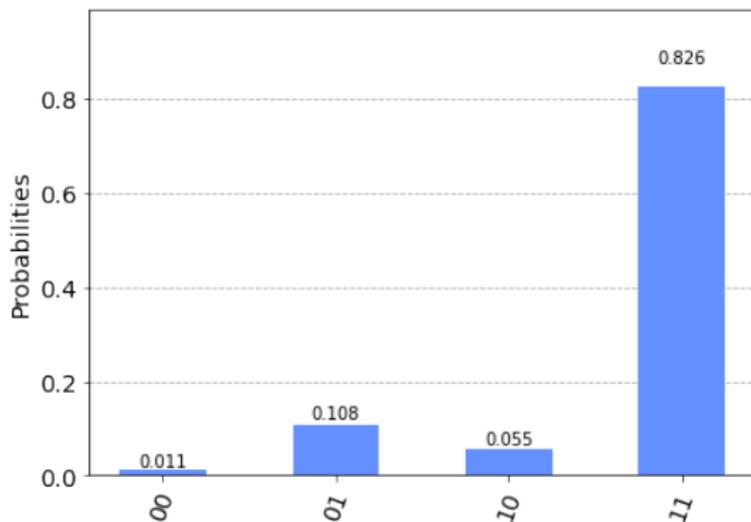
Perfect results! Let's run it for real!

Amplitude Amplification in Practice: Calling IBMQ

Now, running our circuit on IBMQ-Lima, a real 5-qubit quantum computer:

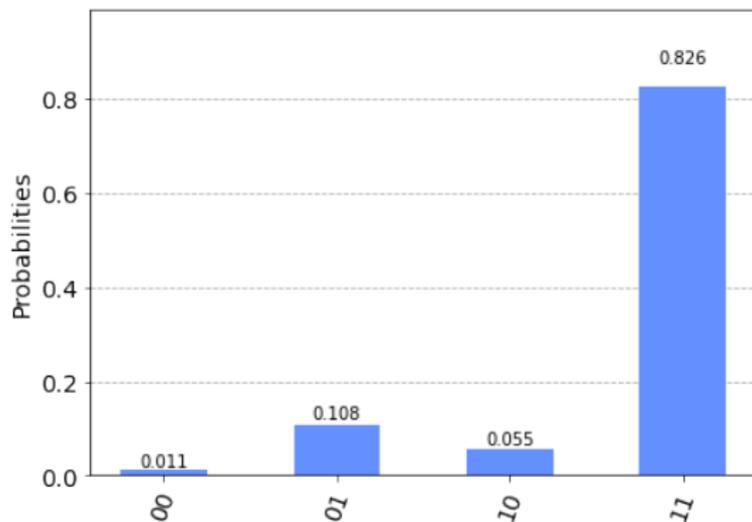
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Amplitude Amplification in Practice: Calling IBMQ

Now, running our circuit on IBMQ-Lima, a real 5-qubit quantum computer:



Not too bad!

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Basic Quantum Algorithms

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- ▶ Quantum Fourier Transform

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Wrapping Up!

Basic Quantum Algorithms

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Qiskit

- ▶ Visualize qubit evolution
- ▶ Simulate experiments on circuits
- ▶ Run circuits on real computers!

Moving Forward

Where to go from here?

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- ▶ Build to more complex applications

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- ▶ Extend to quantum machine learning

Moving Forward

Where to go from here?

- ▶ Build to more complex applications
- ▶ Extend to quantum machine learning
- ▶ Run some interesting experiments

That's it!

Notebooks for the circuits run here can be found at
<https://alexheilman.com>

References I

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Thanks