QIS/QCS Seminar

Meeting 2

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February 6, 2023

Overview

- Criteria for real qubits

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- Criteria for real qubits, what do we want?

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- Finite dimensional

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- Finite dimensional, discernible

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- Example systems

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- Finite dimensional, discernible, evolvable, measurable, reasonably stable
- Example systems: QHO, NMR

Aside: A Note on Decoherence I

There's often talk of decoherence times and the general tendency for quantum states to decohere. An intuitive heuristic for decoherence is possible in light of the general map of operations given last time.

Aside: A Note on Decoherence II

Suppose we have some subsystem of interest with some initial state ψ as below.

$$|\psi\rangle\langle\psi| = \sum_{ij} |i\rangle\langle i|\psi\rangle\langle\psi|j\rangle\langle j| = \sum_{ij} \psi_i\psi_j^* |i\rangle\langle j|$$

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The decoherent state then is the following:

$$|\psi_D\rangle\langle\psi_D| = \sum_i \psi_i^*\psi_i|i\rangle\langle i| = \sum_i p_i|i\rangle\langle i|$$

which is exactly a classical probability distribution over it's previously possible states (think mixed state ensemble).

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Vanishing of Off Diagonal Terms: The above may be derived from a quantum operation of the form:

$$\rho \to \rho_D \mathrm{Tr}_{Env}\left(\sum_{i,j} \psi_i\psi_j^* |i\rangle\langle j| \otimes |\epsilon_i\rangle\langle\epsilon_j|\right) = \sum_{i,j} \psi_i\psi_j^* |i\rangle\langle j|\delta_{ij}$$

where the original state interacts in concert with a larger environment and a theorem of Erich Joos and H. D. Zeh (1985) states that the trace over the environment, with the assumption of einselection (environmentally-aligned measurements) essentially removes the off-diagonal terms of the reduced density matrix.

Aside: A Note on Decoherence III

The probability of measuring the original state to be in some other state ϕ (after projecting both to a common set of basis states)is given by the expression below:

$$\begin{aligned} P(\phi|\psi) &= \langle\phi|\psi\rangle\langle\psi|\phi\rangle \\ &= \sum_{ij}\langle\phi|i\rangle\langle i|\psi\rangle\langle\psi|j\rangle\langle j|\phi\rangle \\ &= \sum_{ij}\phi_i^*\psi_i\psi_j^*\phi_j \end{aligned}$$

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After complete decoherence, the off-diagonal terms disappear, hence no interference terms are left and the probability now becomes that below.

$$\begin{aligned} P(\phi|\psi_D) &= \sum_i \psi_i \psi_i^* \langle\phi|i\rangle\langle i|\phi\rangle \\ &= \sum_i \phi_i^* \psi_i \psi_i^* \phi_i \end{aligned}$$

What do we want?

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- Discrete system

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- Preparable, in a known state

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Then, of course there's the practical matters: we need it to be relatively noise-less, robust, and have a reasonable computation time (relative it's time of expectable stability)

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Weak measurements are also a possibility

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As long as we can get one initial state for sure, we can turn it into an arbitrary state with enough control over it's evolution......

Aside: Thermal State

Recall (from statistical mechanics) that a quantum system at some temperature T is the mixed state:

$$\rho(T) = \frac{1}{N} e^{-E_n/K_B T} |\psi_n\rangle\langle\psi_n| \qquad w/N = \mathrm{Tr}(e^{-E_n/K_B T} |\psi_n\rangle\langle\psi_n|)$$

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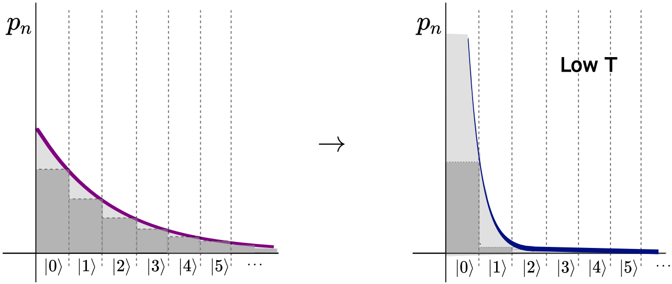

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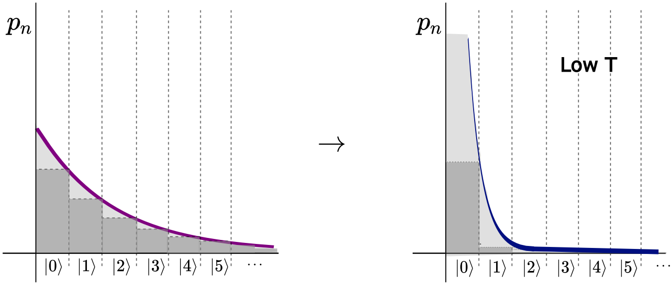

Thus, if we have low enough temperatures, we may essentially initialize some systems.

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We know that Hamiltonians are Hermitian operators and thus are formed a basis by the Pauli matrices in two dimensions:

$$H = a_0 \mathbb{I} + \sum_{i=1}^{3} a_i \sigma^i$$

Thus, if we can control the coefficients of the Pauli matrices a_i independent of each other and by some continuously varying physical means, we control the entire qubit space.

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But what about multi-qubit spaces?

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So, for multi-qubit operation we just need to establish control over Hamiltonian coefficients of one qubit space (really only two Pauli coefficients) + multi-qubit control gates (like CNOT).

(Other basis are also possible and common)

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Obviously, we would also like to minimize resource requirements for operation.

Other Criteria

DiVincenzo's Criteria: The first five are for quantum computers.

- Scalable and discernible
- Fiduciary initial state
- Long decoherence time
- Universal gate set
- Measureable

These next two are for quantum communication

- Memory $\rightarrow$ Computation
- Faithful transmission

Aside: Who has what?

IBM - Superconducting Transmon Qubits

Google - Superconducting Transmon Qubits

Microsoft - Topological Qubits

D-Wave - Annealment

Basic Examples of Qubit Systems

Let's look at some examples!

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Define realizable qubit states

Tunable Hamiltonian for control

Show CNOT gate is possible

Example: Quantum Harmonic Oscillator

Ladder operator form of QHO Hamiltonian:

$$H = \hbar\omega(\frac{1}{2} + a^{\dagger}a)$$

Corresponding eigenvalues denoted $|n\rangle$

$$H|n\rangle = \hbar\omega(\frac{1}{2} + n)|n\rangle = E_n|n\rangle$$

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which evolves (according to $U = e^{-iHt/\hbar}$) as:

$$|\psi(t)\rangle = \sum_{n} e^{-in\omega t} c_n|n\rangle$$

Example: QHO

Example: As an example of a 'two-qubit' gate, we construct a CNOT gate for the QHO. First, imagine we construct the computational basis from the first four energy levels of some QHO.

$$|00\rangle_C = |0\rangle_{QHO}$$
$$|01\rangle_C = |2\rangle_{QHO}$$
$$|10\rangle_C = \frac{1}{\sqrt{2}}(|4\rangle + |1\rangle)_{QHO}$$
$$|11\rangle_C = \frac{1}{\sqrt{2}}(|4\rangle - |1\rangle)_{QHO}$$

where subscript C or QHO denotes the computational basis or QHO energy state basis. Now, considering the action of the Hamiltonian, on the timescale $t = \pi/\omega$ ($U = e^{-i\pi n}$), the Hamiltonian has the effect on the computational basis states:

$$U|00\rangle_C = e^0|0\rangle_{QHO} = |00\rangle \quad U|01\rangle = e^{-i\pi\cdot 2}|2\rangle = |01\rangle$$

$$U|10\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\pi\cdot 4}|4\rangle + e^{-i\pi\cdot 1}|1\rangle\right) = |11\rangle$$

$$U|11\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\pi\cdot 4}|4\rangle - e^{-i\pi\cdot 1}|1\rangle\right) = |10\rangle$$

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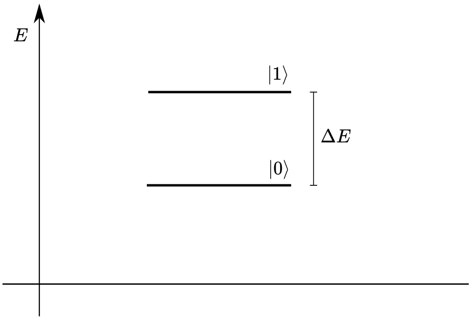
$$E = (n + \frac{1}{2})\hbar\omega$$

It's not discernible! Different states have the same energy difference, so it's hard to tell what happened based upon, e.g., photo-emission, and hard to control excitations.

An intrinsic two-level system

What we'd ideally have for a qubit is a two-level system, which naturally should have some gap in energy (i.e. non-degenerate two-level system).



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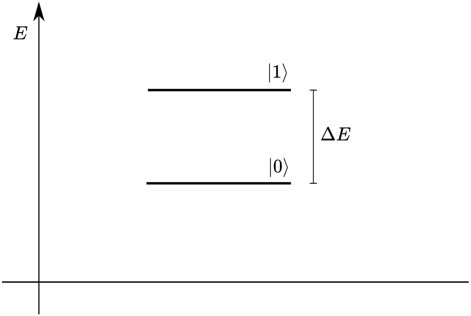

This may be accomplished with a spin 1/2 system in a magnetic field! (Up or down alignment)

Magnetic Field - Spin Interaction

We may describe the Hamiltonian term corresponding to the magnetic field $\leftrightarrow$ spin coupling with the following expression:

$$H_{int} = -\vec{\mu} \cdot \vec{B}$$

Where $\vec{\mu} = \frac{\gamma\hbar}{2}(\sigma_x, \sigma_y, \sigma_z)$ is the analagous magnetic dipole vector of the spin, and γ is the gyromagetic ratio (which is unique to every molecule/atom).

B - μ Interaction Term: $\vec{B} = B\hat{z}$

For a constant magnetic field in the $\hat{z}$ direction, we then have the following form of the interaction Hamiltonian:

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Note that $|0\rangle$ and $|1\rangle$ are eigenstates of σ_z with respective eigenvalues $\lambda = 1, -1$. Thus, the above may be simplified to

$$|\psi(t)\rangle = e^{i\gamma Bt/2}\alpha|0\rangle + e^{-i\gamma Bt/2}\beta|1\rangle$$

B - μ Interaction: Larmor Frequency

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$$|\psi(t)\rangle = e^{i\omega_0 t/2}\alpha|0\rangle + e^{-i\omega_0 t/2}\beta|1\rangle$$

This can be interpreted as a rotation in the X-Y plane of the Bloch sphere, like the precession of a classical top.

$$\langle X\rangle = \cos(\omega_0 t) \qquad \langle Y\rangle = \sin(\omega_0 t)$$

B - μ Interaction: Energy Levels

Note that our form of the interaction Hamiltonian naturally gives us two energy levels for the two states $|0\rangle$ and $|1\rangle$.

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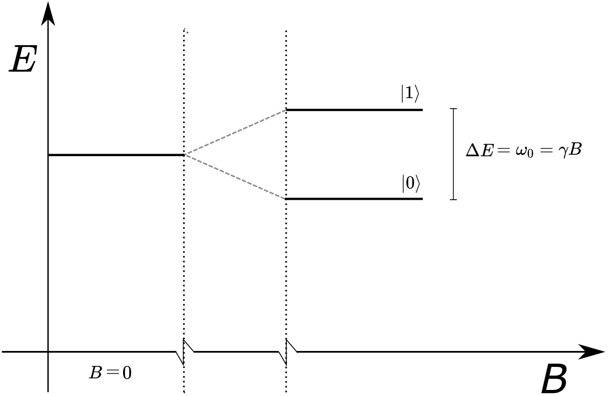
(Zeeman Effect)



B - μ Interaction: Rotating Magnetic Field

Now, consider a more general magnetic field of the form below;

$$\vec{B}(t) = B_0\hat{z} + B_1(t)\left[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right]$$

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If we then define some frame of reference (denoted by superscript *RF*) in which we spin about the z-axis at a rate ω_0, we can clearly see the control we have over the single-qubit Hamiltonian:

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But, what about two qubits?

System: Nuclear Magnetic Resonance

For a coupled two spin-1/2 system in a constant magnetic field in the z-direction, we may model the spin - spin interaction as below, in the weak coupling limit:

$$H_{int} = \frac{\omega_1}{2}(\sigma_z \otimes \mathbb{I}) + \frac{\omega_2}{2}(\mathbb{I} \otimes \sigma_z) + \frac{\pi}{2} J_{12}(\sigma_z \otimes \sigma_z)$$

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(This results from second-order perturbation theory applied to the more general Heisenberg spin-spin coupling $\propto \sigma_1 \cdot \sigma_2$)

Note that, in general, the two spin-1/2 systems have a different gyromagnetic ratio and hence different Larmor frequencies ω_1, ω_2.

In general, this interaction between the spins is known as J-coupling or Fermi-type interaction.

System: NMR

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Can explore in more detail if interest is there.

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Recap

Hope you still got something out of it! Criteria for QC systems:

- Discernible, persistent, discrete states (qubits)
- Reliably initializable in some known state
- Tunable control over portion of Hamiltonian acting on one and two-qubit states
- Appropriate, associated measurement apparatus

Future?

- Quantum mechanics as probability theory?
- Common circuits?
- Lasers and coherent states?
- Josephson junctions, transmon paper, NMR in more detail?