Quantum Teleportation

No-cloning and a simple example

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Overview

- Quantum No-Cloning

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- Quantum Teleportation

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- Quantum Teleportation (Two-Party, Qubit State)

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- Experimental Setup

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NOT ALLOWED!!!

No-Cloning: Simple Proof

Consider the following action of a 'copy' on the basis states:

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CONTRADICTION!

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A more general theorem that extends to mixed states and quantum operations is known as the quantum no-broadcasting theorem.

Quantum Teleportation

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Often times, party one is termed Alice (A) and party two is termed Bob (B)

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State is now 'teleported'!

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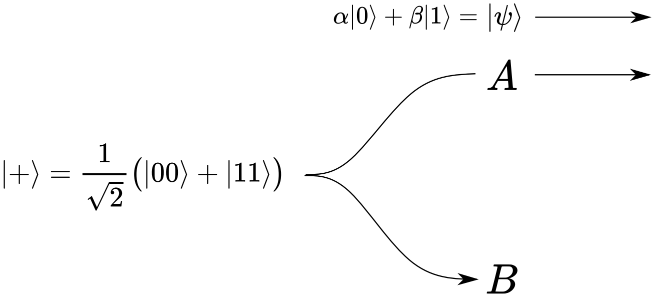
The necessary transmission of the (classical) information describing the measurement outcome of party A guarantees that causality is preserved.

Teleportation 1

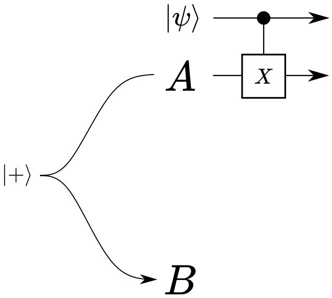
Alice begins with state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and receives one half of the entangled Bell pair $|+\rangle = \big(|00\rangle + |11\rangle\big)/\sqrt{2}$



$$\frac{1}{\sqrt{2}}\left[\alpha(|000\rangle + |011\rangle) + \beta(|100\rangle + |111\rangle)\right]$$

Teleportation 2 (a)

Alice applies a controlled X gate on her qubit state (as control) and her half of the entangled pair (as target)



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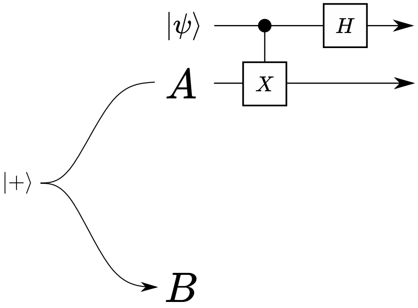
Teleportation 2 (b)

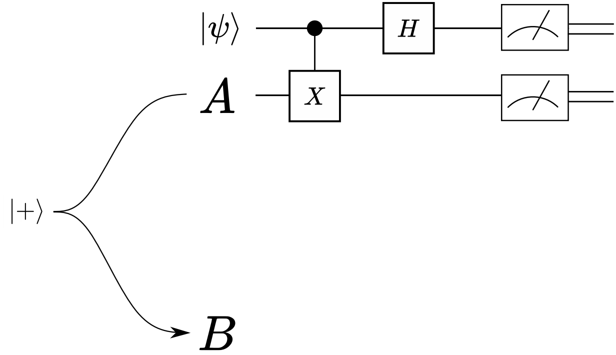
Alice then applies a Hadamard (H) gate on her qubit state



$$\frac{1}{2}\Big[\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) \\ +\beta(|001\rangle + |010\rangle - |101\rangle - |110\rangle)\Big]$$

Teleportation 3

Alice then measures her qubit and her half of the entangled qubit



Teleportation 4

Recall that the total state before measurement is proportional to the following:

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Let's now consider what such measurement results tell us about the resulting state (which Bob is in possession of):

$$\begin{aligned}\text{A measures}\quad |00\rangle \rightarrow &\quad \alpha|0\rangle + \beta|1\rangle = |\psi\rangle \\ |01\rangle \rightarrow &\quad \alpha|1\rangle + \beta|0\rangle = X|\psi\rangle \\ |10\rangle \rightarrow &\quad \alpha|0\rangle - \beta|1\rangle = Z|\psi\rangle \\ |11\rangle \rightarrow &\quad \alpha|1\rangle - \beta|0\rangle = XZ|\psi\rangle\end{aligned}$$

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So, Alice may transmit her measurement results (classically) to Bob, and he will know exactly what state he has with regards to Alice's arbitrary initial state.

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Bob is then guaranteed to have the qubit state $|\psi\rangle$!

Notes on Teleportation

Note that Alice's state has been destroyed but she has effectively transmitted it or 'teleported' it to Bob. In fact, Alice may not even know the state of her qubit, $|\psi\rangle$.

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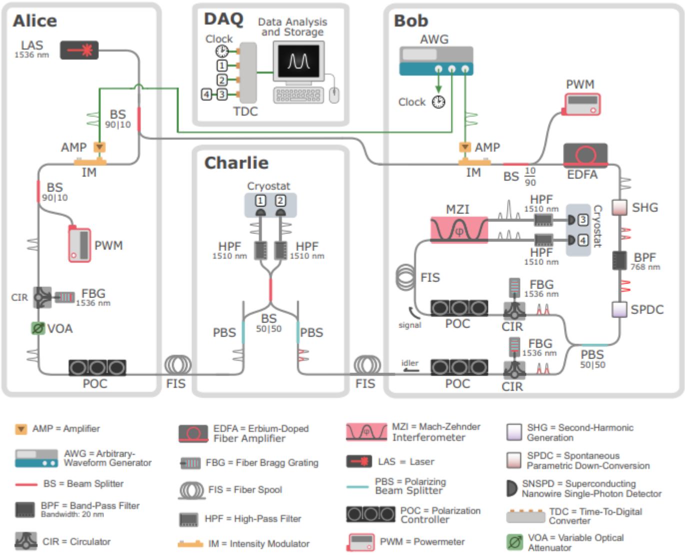
Hence, quantum teleportation can help reduce computational errors, be used to construct more resilient quantum networks, and form secure communication channels. May be the foundation of any future 'quantum internet'!

Recent experiment



[1]

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Such schemes may be the foundation for future quantum networks

Next time?

Any ideas?

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Any ideas? Thanks for your time!

References I

Raju Valivarthi, Samantha I. Davis, Cristián Peña, Si Xie, Nikolai Lauk, Lautaro Narváez, Jason P. Allmaras, Andrew D. Beyer, Yewon Gim, Meraj Hussein, George Iskander, Hyunseong Linus Kim, Boris Korzh, Andrew Mueller, Mandy Rominsky, Matthew Shaw, Dawn Tang, Emma E. Wollman, Christoph Simon, Panagiotis Spentzouris, Daniel Oblak, Neil Sinclair, and Maria Spiropulu.
Teleportation systems toward a quantum internet.
PRX Quantum, 1:020317, Dec 2020.